Maths | Intent and Implementation

Why do we teach maths?

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems.

It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment.

A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

What is our curriculum aim?

The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

Maths | Intent and Implementation

How is maths taught?

Maths is taught using a 'mastery' approach. Mastering maths means pupils acquiring a deep, long-term, secure and adaptable understanding of the subject and being able to apply concepts in many different contexts. Maths is taught in mixed-ability class groups, where the focus is on all pupils working together on the same lesson content at the same time, as happens in Shanghai and several other regions that teach maths successfully. This ensures that all can master concepts before moving to the next part of the curriculum sequence, allowing no pupil to be left behind. If a pupil fails to grasp a concept or procedure, this is identified quickly, and early intervention ensures the pupil is ready to move forward with the whole class.

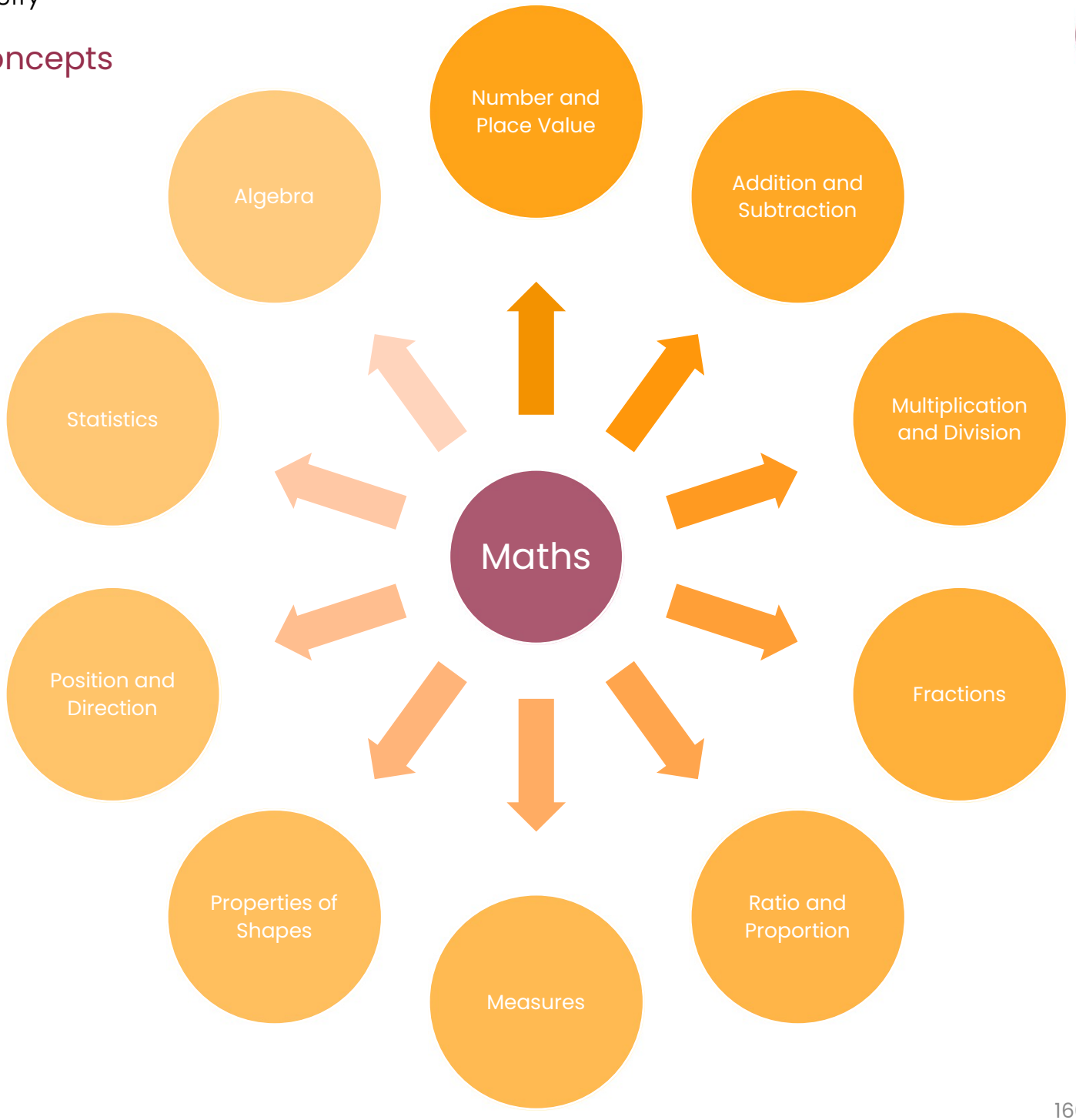
Teaching is based on the Maths Hub approach, with lesson design identifying the new mathematics that is to be taught, the key points and potential misconceptions to create a carefully sequenced journey through the learning. Procedural fluency and conceptual understanding are developed in tandem because each supports the development of the other.

Discussion is a key part of teaching, with children being expected to explain their approach to questions; this allows for the development of deeper understanding as well as providing assessment opportunities. Each lesson follows the 'I do, We do, You do' approach with teacher models of the concept being followed by shared work before independent 'Intelligent Practice' that both reinforces pupils' procedural fluency and develops their conceptual understanding.

Concrete resources are available in all classrooms, with the expectation that children will move from the use of these through pictorial representations to abstract as they gain a secure mental model of the concept. Throughout school children are introduced to a range of concrete resources and are strongly encouraged to use these to develop a deeper understanding of concepts by seeing it visually, rather than as an abstract.

Daily fluency sessions happen outside of the main maths lessons, and focus on key facts such as multiplication tables and addition facts. In Key Stage 1 children use Mastering Number which focusses on key facts which are learnt to automaticity to avoid cognitive overload in the working memory and enable pupils to focus on new concepts. In KS2 children continue to learn multiplication facts and the structure of multiplication through Mastering Number KS2.

Maths | Key Concepts



Maths | Content Spine

	Autumn	Spring	Summer
Y1	• Counting, recognising & comparing no.s 0 – 10 • Counting to and from 20 • Counting in tens - decade numbers • Pattern in counting from 20 to 100 • Comparing quantities - part whole relationships • Comparing quantities - part whole relationships	• Composition of numbers 0 to 5 • Recognise, compose, decompose and manipulate 2D and 3D shapes • Composition of numbers 6 to 10 • Additive structures: addition • Additive structures: addition and subtraction • Addition and subtraction facts within 10	• Composition of numbers 11 to 19 • Numbers 0 to 20 in different contexts • Unitising and coin recognition - counting in 2s, 5s and 10s • Unitising and coin recognition - value of a set of coins • Solving problems in a range of contexts • Position and direction including fractions of turns • Time - sequencing events and telling the time to the hour and half hour
Y2	• Composition of multiples of 10 • Counting and representing the numbers 20 to 99 • Comparing, ordering and partitioning 2-digit numbers • Secure fluency of addition and subtraction facts within 10 • Calculating within 20 • Adding and subtracting ones and tens to and from 2-digit numbers	• Grouping objects in different ways and relating to multiplication • Representing counting in 2s, 5s and 10s as the 2, 5 and 10 times tables • Representing counting in 5s as the 5 times table and link to the 10 times tables • Multiplying by 2, doubling and halving (factors and products) • Introduction to division structures • Shape: discuss and compare 2D and 3D shapes	• Addition and subtraction of two 2-digit numbers • Money: recognise coins and use £ and p symbols • Fractions: identify equal parts and be familiar with halves, thirds and quarters • Time: write and tell the time to five minutes • Position and direction • Doubling, halving, quotative and partitive division • Sense of measure - capacity, volume and mass
Y3	• Review strategies for adding and subtracting across 10 • Securing place value to 100 and applying to addition and subtraction • Bridging 100: counting on and back in 10s, adding/subtracting multiples of 10 • Measuring length and recording in tables • Representing 3-digit numbers, comparing and positioning on number lines • Measures: mass and capacity	• Right angles • Informal and mental strategies for adding and subtracting two 3-digit numbers • Understand additive relationships and apply them to rearrange equations • Column addition • 2, 4 and 8 times tables: using times tables to solve problems • Column subtraction	• Unit fractions as part of a whole • Identify parts and wholes in different contexts • Compare and order unit fractions • Calculate the value of a part (fractions as operators) • Non-unit fractions • Composition of non-unit fractions: addition and subtraction • Parallel and perpendicular sides in polygons (and perimeter) • Tell the time to the nearest minute and compare units of time

Our Curriculum: Core Faculty

Maths | Content Spine

Y4	• Review of column addition and subtraction • Secure place value to 1000: apply to addition and subtraction: multiples of 100 • Calculation and conversion of measures • Comparing, ordering and rounding 4-digit numbers • Column addition and subtraction with 4-digit numbers • Perimeter • Represent counting in threes and sixes as the 3 and 6 times tables • Relationship between the 3 and 6 times tables and tests of divisibility	• Represent counting in nines as the 9 times table • Relationship between the 3 and 9 times tables • 7 times table: odd and even patterns, square numbers and tests of divisibility • Understand and represent multiplicative structures • Apply the distributive law to multiplication • Understand what happens when a number is multiplied or divided by 10 and 100 • Coordinates	• Review of fractions • Composition of fractions greater than one • Compare and order mixed numbers and position on a number line • Addition and subtraction of fractions and mixed numbers (within a whole) • Convert improper fractions to mixed numbers and vice versa • Efficient strategies for adding and subtracting mixed numbers (crossing a whole) • Properties of 2D and 3D shapes and symmetry • Time: Convert between 12 and 24 hour clocks: analogue and digital • Division with remainders
Y5	• Understand tenths as part of a whole, represent and calculate mentally • Compose and calculate with decimals including column addition and subtraction • Understand hundredths as parts of a whole and represent • Use knowledge of decimals to solve problems in different contexts: length • Money: apply efficient strategies when calculating with money • Negative numbers • Multiplication by partitioning leading to short multiplication (2 by 1-digit) • Multiplication by partitioning leading to short multiplication (3 by 1-digit)	• Division by partitioning leading to short division (2 and 3-digits by 1-digit) • Understand the concept of area • Link area of rectangles to multiplication • Compare and describe measurements using knowledge of multiplication and division • Calculating with decimal fractions • Understand the concept of volume • Multiply 3 or more numbers (commutative and associative laws) • Understand and use the concept of factorisation (square and prime numbers) • Use common factors and multiples to solve calculations efficiently	• Multiply a proper fraction by a whole number • Multiply improper fractions and mixed numbers by a whole number • Find unit and non-unit fractions of whole numbers exploring parts and wholes • Comparing fractions using equivalence and decimals • Converting units • Angles: compare, name, estimate and measure angles
Y6	• Angles: compare, name, estimate and measure angles • Use equivalence and compensation to simplify and solve addition calculations • Use equivalence and compensation to simplify and solve subtraction problems • Multiples of 1,000 • Understand place value within numbers with up to 7 digits • Draw, compose and decompose shapes • Using equivalence to calculate	• Multiplying and dividing by 2-digit numbers • Area, perimeter, position and direction • Addition and subtraction of fractions • Comparing fractions • Multiplication and division of fractions • Understanding percentages • Statistics • Ratio and proportion • Calculating using knowledge of equivalence in addition and subtraction	• Solving problems with two unknowns • Order of operations • Mean average

Maths | Progression Map: Number and Place Value

COUNTING					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number			count backwards through zero to include negative numbers	interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero	use negative numbers in context, and calculate intervals across zero
count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens	count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward	count from 0 in multiples of 4, 8, 50 and 100;	count in multiples of 6, 7, 9, 25 and 1000	count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000	
given a number, identify one more and one less		find 10 or 100 more or less than a given number	find 1000 more or less than a given number		
COMPARING NUMBERS					
use the language of: equal to, more than, less than (fewer), most, least	compare and order numbers from 0 up to 100; use <, > and = signs	compare and order numbers up to 1000	order and compare numbers beyond 1000 compare numbers with the same number of decimal places up to two decimal places (copied from Fractions)	read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit (appears also in Reading and Writing Numbers)	read, write, order and compare numbers up to 10 000 000 and determine the value of each digit (appears also in Reading and Writing Numbers)
IDENTIFYING, REPRESENTING AND ESTIMATING NUMBERS					
identify and represent numbers using objects and pictorial representations including the number line	identify, represent and estimate numbers using different representations, including the number line	identify, represent and estimate numbers using different representations	identify, represent and estimate numbers using different representations		

Our Curriculum: Core Faculty

Maths | Progression Map: Number and Place Value

READING AND WRITING NUMBERS (including Roman Numerals)					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
read and write numbers from 1 to 20 in numerals and words.	read and write numbers to at least 100 in numerals and in words	read and write numbers up to 1000 in numerals and in words		read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit (appears also in Comparing Numbers)	read, write, order and compare numbers up to 10 000 000 and determine the value of each digit (appears also in Understanding Place Value)
		tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks (copied from Measurement)	read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.	read Roman numerals to 1000 (M) and recognise years written in Roman numerals.	
UNDERSTANDING PLACE VALUE					
	recognise the place value of each digit in a two-digit number (tens, ones)	recognise the place value of each digit in a three-digit number (hundreds, tens, ones)	recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)	read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit (appears also in Reading and Writing Numbers)	read, write, order and compare numbers up to 10 000 000 and determine the value of each digit (appears also in Reading and Writing Numbers)
			find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths (copied from Fractions)	recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents (copied from Fractions)	identify the value of each digit to three decimal places and multiply and divide numbers by 10, 100 and 1000 where the answers are up to three decimal places (copied from Fractions)

Maths | Progression Map: Number and Place Value

ROUNDING					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
			round any number to the nearest 10, 100 or 1 000	round any number up to 1 000 000 to the nearest 10, 100, 1 000, 10 000 and 100 000	round any whole number to a required degree of accuracy
			round decimals with one decimal place to the nearest whole number (copied from Fractions)	round decimals with two decimal places to the nearest whole number and to one decimal place (copied from Fractions)	solve problems which require answers to be rounded to specified degrees of accuracy (copied from Fractions)
PROBLEM SOLVING					
	use place value and number facts to solve problems	solve number problems and practical problems involving these ideas.	solve number and practical problems that involve all of the above and with increasingly large positive numbers	solve number problems and practical problems that involve all of the above	solve number and practical problems that involve all of the above

Our Curriculum: Core Faculty

Maths | Progression Map: Number – Addition and Subtraction

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
NUMBER BONDS					
represent and use number bonds and related subtraction facts within 20	recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100				
MENTAL CALCULATION					
add and subtract one-digit and two-digit numbers to 20, including zero	add and subtract numbers using concrete objects, pictorial representations, and mentally, including: • a two-digit number and ones • a two-digit number and tens • two two-digit numbers • adding three one-digit numbers	add and subtract numbers mentally, including: • a three-digit number and ones • a three-digit number and tens • a three-digit number and hundreds		add and subtract numbers mentally with increasingly large numbers	perform mental calculations, including with mixed operations and large numbers
read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs (appears also in Written Methods)	show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot				use their knowledge of the order of operations to carry out calculations involving the four operations
WRITTEN METHODS					
read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs (appears also in Mental Calculation)		add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	

Maths | Progression Map: Number – Addition and Subtraction

INVERSE OPERATIONS, ESTIMATING AND CHECKING ANSWERS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.	estimate the answer to a calculation and use inverse operations to check answers	estimate and use inverse operations to check answers to a calculation	use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy	use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
PROBLEM SOLVING					
solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = □ - 9	solve problems with addition and subtraction: * using concrete objects and pictorial representations, including those involving numbers, quantities and measures * applying their increasing knowledge of mental and written methods	solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction	solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why	solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
	solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change (copied from Measurement				Solve problems involving addition, subtraction, multiplication and division

Maths | Progression Map: Number – Multiplication and Division

MULTIPLICATION & DIVISION FACTS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
count in multiples of twos, fives and tens (copied from Number and Place Value)	count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward (copied from Number and Place Value)	count from 0 in multiples of 4, 8, 50 and 100 (copied from Number and Place Value)	count in multiples of 6, 7, 9, 25 and 1000 (copied from Number and Place Value)	count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 (copied from Number and Place Value)	
	recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers	recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	recall multiplication and division facts for multiplication tables up to 12 × 12		
MENTAL CALCULATION					
		write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods (appears also in Written Methods)	use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers	multiply and divide numbers mentally drawing upon known facts	perform mental calculations, including with mixed operations and large numbers
	show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot		recognise and use factor pairs and commutativity in mental calculations (appears also in Properties of Numbers)	multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $^3/_8$) (copied from Fractions)

Our Curriculum: Core Faculty

Maths | Progression Map: Number – Multiplication and Division

WRITTEN CALCULATION					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs	write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods (appears also in Mental Methods)	multiply two-digit and three-digit numbers by a one-digit number using formal written layout	multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
				divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
					use written division methods in cases where the answer has up to two decimal places (copied from Fractions (including decimals))

Maths | Progression Map: Number – Multiplication and Division

PROPERTIES OF NUMBERS: MULTIPLES, FACTORS, PRIMES, SQUARE AND CUBE NUMBERS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
			recognise and use factor pairs and commutativity in mental calculations (repeated)	identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers establish whether a number up to 100 is prime and recall prime numbers up to 19	identify common factors, common multiples and prime numbers *use common factors to simplify fractions; use common multiples to express fractions in the same denomination* (copied from Fractions)
				recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})	*calculate, estimate and compare volume of cubes and cuboids using standard units, including centimetre cubed (cm^3) and cubic metres (m^3), and extending to other units such as mm^3 and km^3* (copied from Measures)
ORDER OF OPERATIONS					
					use their knowledge of the order of operations to carry out calculations involving the four operations

Maths | Progression Map: Number – Multiplication and Division

NUM INVERSE OPERATIONS, ESTIMATING AND CHECKING ANSWERSBER BONDS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
		estimate the answer to a calculation and use inverse operations to check answers (copied from Addition and Subtraction)	*estimate and use inverse operations to check answers to a calculation* (copied from Addition and Subtraction)		use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy
PROBLEM SOLVING					
solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts	solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects	solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects	solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes	solve problems involving addition, subtraction, multiplication and division
				solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign	
				solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates	*solve problems involving similar shapes where the scale factor is known or can be found* (copied from Ratio and Proportion)

Our Curriculum: Core Faculty

Maths | Progression Map: Fractions (including Decimals and Percentages)

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
COUNTING IN FRACTIONAL STEPS					
	Pupils should count in fractions up to 10, starting from any number and using the1/2 and 2/4 equivalence on the number line (Non Statutory Guidance)	count up and down in tenths	count up and down in hundredths		
RECOGNISING FRACTIONS					
recognise, find and name a half as one of two equal parts of an object, shape or quantity recognise, find and name a quarter as one of four equal parts of an object, shape or quantity	recognise, find, name and write fractions $^1/_3$, $^1/_4$, $^2/_4$ and $^3/_4$ of a length, shape, set of objects or quantity	recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators recognise that tenths arise from dividing an object into 10 equal parts and in dividing one – digit numbers or quantities by 10. recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators	recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten	recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents (appears also in Equivalence)	
COMPARING FRACTIONS					
		compare and order unit fractions, and fractions with the same denominators		compare and order fractions whose denominators are all multiples of the same number	compare and order fractions, including fractions >1
			compare numbers with the same number of decimal places up to two decimal places	read, write, order and compare numbers with up to three decimal places	identify the value of each digit in numbers given to three decimal places

Maths | Progression Map: Fractions – Multiplication and Division

ROUNDING INCLUDING DECIMALS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
			round decimals with one decimal place to the nearest whole number	round decimals with two decimal places to the nearest whole number and to one decimal place	solve problems which require answers to be rounded to specified degrees of accuracy
EQUIVALENCE (INCLUDING FRACTIONS, DECIMALS AND PERCENTAGES)					
	write simple fractions e.g. $^1/_2$ of 6 = 3 and recognise the equivalence of $^2/_4$ and $^1/_2$.	recognise and show, using diagrams, equivalent fractions with small denominators	recognise and show, using diagrams, families of common equivalent fractions	identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths	use common factors to simplify fractions; use common multiples to express fractions in the same denomination
			recognise and write decimal equivalents of any number of tenths or hundredths	read and write decimal numbers as fractions (e.g. 0.71 = $^{71}/_{100}$) recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents	associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $^3/_8$)
			recognise and write decimal equivalents to $^1/_4$; $^1/_2$; $^3/_4$	recognise the per cent symbol (%) and understand that per cent relates to "number of parts per hundred", and write percentages as a fraction with denominator 100 as a decimal fraction	recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.

Maths | Progression Map: Fractions – Multiplication and Division

ADDITION AND SUBTRACTION OF FRACTIONS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
		add and subtract fractions with the same denominator within one whole (e.g. $^5/_7 + {}^1/_7 = {}^6/_7$)	add and subtract fractions with the same denominator	add and subtract fractions with the same denominator and multiples of the same number recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number (e.g. $^2/_5 + {}^4/_5 = {}^6/_5 = 1{}^1/_5$)	add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
MULTIPLICATION AND DIVISION OF FRACTIONS					
				multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams	multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $^1/_4 \times {}^1/_2 = {}^1/_8$) multiply one-digit numbers with up to two decimal places by whole numbers divide proper fractions by whole numbers (e.g. $^1/_3 \div 2 = {}^1/_6$)

Maths | Progression Map: Fractions – Multiplication and Division

MULTIPLICATION AND DIVISION OF DECIMALS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
					multiply one-digit numbers with up to two decimal places by whole numbers
			find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths		multiply and divide numbers by 10, 100 and 1000 where the answers are up to three decimal places
					identify the value of each digit to three decimal places and multiply and divide numbers by 10, 100 and 1000 where the answers are up to three decimal places
					associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $^{3}/_{8}$)
					use written division methods in cases where the answer has up to two decimal places

Maths | Progression Map: Fractions – Multiplication and Division

PROBLEM SOLVING					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
		solve problems that involve all of the above	solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number	solve problems involving numbers up to three decimal places	
			solve simple measure and money problems involving fractions and decimals to two decimal places.	solve problems which require knowing percentage and decimal equivalents of $^1/_2$, $^1/_4$, $^1/_5$, $^2/_5$, $^4/_5$ and those with a denominator of a multiple of 10 or 25.	

Maths | Progression Map: Ratio and Proportion

NUMBER					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
					solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
					solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison
					solve problems involving similar shapes where the scale factor is known or can be found
					solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

Maths | Progression Map: Algebra

EQUATIONS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and ***missing number problems*** *such as* $7 = \square - 9$ (copied from Addition and Subtraction)	*recognise and use the inverse relationship between addition and subtraction and use this to check calculations and* ***missing number*** *problems.* (copied from Addition and Subtraction)	solve problems, *including* ***missing number*** *problems, using number facts, place value, and more complex addition and subtraction.* (copied from Addition and Subtraction) *solve problems, including* ***missing number*** *problems, involving multiplication and division, including integer scaling* (copied from Multiplication and Division)		*use the properties of rectangles to deduce related facts and find* ***missing lengths and angles*** (copied from Geometry: Properties of Shapes)	express missing number problems algebraically
	recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 (copied from Addition and Subtraction)				find pairs of numbers that satisfy number sentences involving two unknowns
represent and use number bonds and related subtraction facts within 20 (copied from Addition and Subtraction)					enumerate all possibilities of combinations of two variables

Maths | Progression Map: Algebra

FORMULAE					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
			Perimeter can be expressed algebraically as 2(a + b) where a and b are the dimensions in the same unit. *(Copied from NSG measurement)*		use simple formulae *recognise when it is possible to use* ***formulae*** *for area and volume of shapes* (copied from Measurement)
SEQUENCES					
sequence events in chronological order using language such as: before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening (copied from Measurement)	*compare and sequence intervals of time* (copied from Measurement) order and arrange combinations of mathematical objects in patterns (copied from Geometry: position and direction)				generate and describe linear number sequences

Our Curriculum: Core Faculty

Maths | Progression Map: Measurement

COMPARING AND ESTIMATING					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
compare, describe and solve practical problems for: • lengths and heights [e.g. long/short, longer/shorter, tall/short, double/half] • mass/weight [e.g. heavy/light, heavier than, lighter than] • capacity and volume [e.g. full/empty, more than, less than, half, half full, quarter] • time [e.g. quicker, slower, earlier, later]	compare and order lengths, mass, volume/capacity and record the results using >, < and =		estimate, compare and calculate different measures, including money in pounds and pence (also included in Measuring)	calculate and compare the area of squares and rectangles including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes (also included in measuring) estimate volume (e.g. using 1 cm^3 blocks to build cubes and cuboids) and capacity (e.g. using water)	calculate, estimate and compare volume of cubes and cuboids using standard units, including centimetre cubed (cm^3) and cubic metres (m^3), and extending to other units such as mm^3 and km^3.
sequence events in chronological order using language [e.g. before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening]	compare and sequence intervals of time	compare durations of events, for example to calculate the time taken by particular events or tasks			
		estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes, hours and o'clock; use vocabulary such as a.m./p.m., morning, afternoon, noon and midnight (appears also in Telling the Time)			

Our Curriculum: Core Faculty

Maths | Progression Map: Measurement

MEASURING and CALCULATING					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
measure and begin to record the following: • **lengths and heights** • **mass/weight** • **capacity and volume** • **time** (hours, minutes, seconds**)**	choose and use appropriate standard units to estimate and measure **length/height** in any direction (m/cm); **mass** (kg/g); **temperature** (°C); **capacity** (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels	measure, compare, add and subtract: **lengths** (m/cm/mm); **mass** (kg/g); **volume/capacity** (l/ml)	estimate, compare and calculate **different measures,** including **money in pounds and pence** (appears also in Comparing)	use all four operations to solve problems involving measure (e.g. **length, mass, volume, money**) using decimal notation including scaling.	solve problems involving the calculation and conversion of **units of measure**, using decimal notation up to three decimal places where appropriate (appears also in Converting)
		measure the **perimeter** of simple 2-D shapes	measure and calculate the **perimeter** of a rectilinear figure (including squares) in centimetres and metres	measure and calculate the **perimeter** of composite rectilinear shapes in centimetres and metres	recognise that shapes with the same areas can have different **perimeters** and vice versa
recognise and know the value of different denominations of **coins and notes**	recognise and use symbols for pounds **(£) and pence (p)**; combine amounts to make a particular value find different combinations of coins that equal the same amounts of money **solve simple problems** in a practical context involving addition and subtraction of money of the same unit, including giving change	add and subtract amounts of **money** to give change, using both £ and p in practical contexts			
			find the area of rectilinear shapes by counting squares	calculate and compare the area of squares and rectangles including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes *recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})* (copied from Multiplication and Division)	calculate the area of parallelograms and triangles calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [e.g. mm^3 and km^3]. recognise when it is possible to use formulae for area and volume of shapes

Maths | Progression Map: Measurement

TELLING THE TIME					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
tell the time to the hour and half past the hour and draw the hands on a clock face to show these times.	tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times.	tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks	read, write and convert time between analogue and digital 12 and 24-hour clocks (appears also in Converting)		
recognise and use language relating to dates, including days of the week, weeks, months and years	know the number of minutes in an hour and the number of hours in a day. (appears also in Converting)	estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes, hours and o'clock; use vocabulary such as a.m./p.m., morning, afternoon, noon and midnight (appears also in Comparing and Estimating)			
			solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days (appears also in Converting)	solve problems involving converting between units of time	

Maths | Progression Map: Measurement

CONVERTING					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	know the number of minutes in an hour and the number of hours in a day. (appears also in Telling the Time)	know the number of seconds in a minute and the number of days in each month, year and leap year	convert between different units of measure (e.g. kilometre to metre; hour to minute)	convert between different units of metric measure (e.g. kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)	use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places
			read, write and convert time between analogue and digital 12 and 24-hour clocks (appears also in Converting)	solve problems involving converting between units of time	solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate (appears also in Measuring and Calculating)
			solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days (appears also in Telling the Time)	understand and use equivalences between metric units and common imperial units such as inches, pounds and pints	convert between miles and kilometres

Maths | Progression Map: Geometry: Properties of Shapes

IDENTIFYING SHAPES AND THIER PROPERTIES					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
recognise and name common 2-D and 3-D shapes, including: * 2-D shapes [e.g. rectangles (including squares), circles and triangles] * 3-D shapes [e.g. cuboids (including cubes), pyramids and spheres].	identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line		identify lines of symmetry in 2-D shapes presented in different orientations	identify 3-D shapes, including cubes and other cuboids, from 2-D representations	recognise, describe and build simple 3-D shapes, including making nets (appears also in Drawing and Constructing)
	identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces				illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
	identify 2-D shapes on the surface of 3-D shapes, [for example, a circle on a cylinder and a triangle on a pyramid]				
DRAWING AND CONSTRUCTING					
		draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them	complete a simple symmetric figure with respect to a specific line of symmetry	draw given angles, and measure them in degrees (°)	draw 2-D shapes using given dimensions and angles recognise, describe and build simple 3-D shapes, including making nets (appears also in Identifying Shapes and Their Properties)

Maths | Progression Map: Geometry: Properties of Shapes

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
COMPARING AND CLASSIFYING					
	compare and sort common 2-D and 3-D shapes and everyday objects		compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes	use the properties of rectangles to deduce related facts and find missing lengths and angles	compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
				distinguish between regular and irregular polygons based on reasoning about equal sides and angles	
ANGLES					
		recognise angles as a property of shape or a description of a turn		know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles	
		identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle	identify acute and obtuse angles and compare and order angles up to two right angles by size	identify: • angles at a point and one whole turn (total 360°) • angles at a point on a straight line and ½ a turn (total 180°) • other multiples of 90°	recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles
		identify horizontal and vertical lines and pairs of perpendicular and parallel lines			

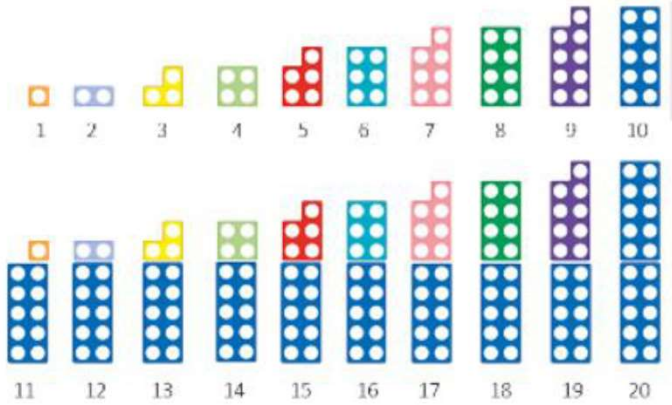
Maths | Progression Map: Geometry: Position and Direction

POSITION, DIRECTION AND MOVEMENT					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
describe position, direction and movement, including half, quarter and three-quarter turns.	use mathematical vocabulary to describe position, direction and movement including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise)		describe positions on a 2-D grid as coordinates in the first quadrant describe movements between positions as translations of a given unit to the left/right and up/down	identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed	describe positions on the full coordinate grid (all four quadrants) draw and translate simple shapes on the coordinate plane, and reflect them in the axes.
			plot specified points and draw sides to complete a given polygon		
PATTERN					
	order and arrange combinations of mathematical objects in patterns and sequences				

Maths | Progression Map: Statistics

INTERPRETING, CONSTRUCTING AND PRESENTING DATA					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	interpret and construct simple pictograms, tally charts, block diagrams and simple tables	interpret and present data using bar charts, pictograms and tables	interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs	complete, read and interpret information in tables, including timetables	interpret and construct pie charts and line graphs and use these to solve problems
	ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity				
	ask and answer questions about totalling and comparing categorical data				
SOLVING PROBLEMS					
		solve one-step and two-step questions [e.g. 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables.	solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.	solve comparison, sum and difference problems using information presented in a line graph	calculate and interpret the mean as an average

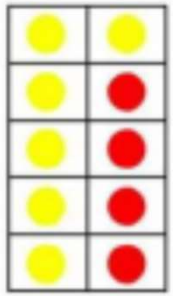
Maths | Modelling - Foundation Stage: Algebra

Reception Number and Number Patterns	
Objective/Strategy	**Concrete and Pictorial Modelling**
Have a deep understanding of number to 10, including the composition of each number.	  
Subitise (recognise quantities without counting) up to 5	 
Numerical Patterns	
Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.	      Can you think of any other ways to represent this number?  

Maths | Modelling - Year 1: Addition

Key Vocabulary

- Sum
- total
- parts and wholes
- Plus
- Add
- altogether
- more
- 'is equal to'
  'is the same as'

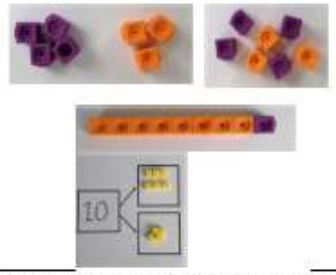
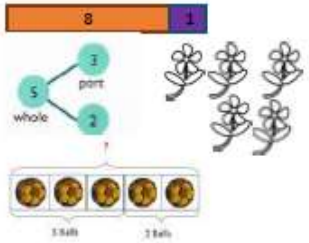
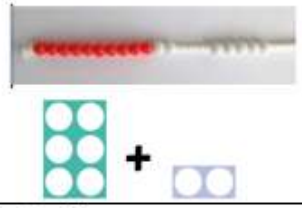
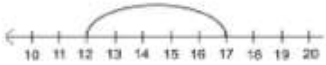


6 + 4 = 10
4 + 6 = 10
10 – 4 = 6
10 – 6 = 4

Year 1 Addition

within 10 and then 20

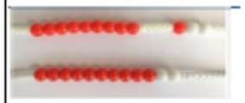
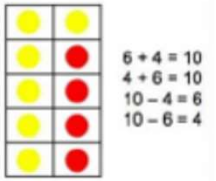
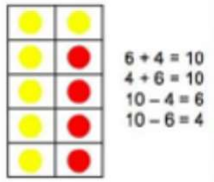
Use dienes or base ten (tens rods and ones/units cubes) for number in preparation for Year 2

Objective/Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model Addition Year 1	Use part-whole model. Use cubes to add two numbers together as a group or in a bar. 	Use pictures to add two numbers together as a group or in a bar. 	Use the part-whole diagram as shown below to move into the abstract. Include missing number questions to support varied fluency. 8 = 5 + 3 5 + 3 = 8  8 = ? + 3 5 + ? = 8
Starting at the bigger number and counting on Addition Year 1	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. Use counters, tens frames or Numicon. 	Start at the larger number on the number line and count on in ones or in one jump to find the answer.  **12** + 5= 17	Place the larger number in your head and count on the smaller number to find your answer. 5 + **12** = 17
Regrouping to make 10 This is an essential skill for column addition later Addition Year 1	6 + 5 = 11 Start with the bigger number and use the smaller number to make 10. Use ten frames. 	Use pictures or a number line. Regroup or partition the smaller number using the part-whole model to make 10. 	7 + 4= 11 3+1 I am at seven, how many more do I need to make 10? (3) How many more do I add on now? (1)

Maths | Modelling - Year 1: Addition continued

Key Vocabulary

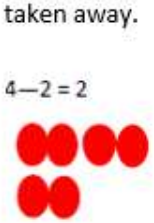
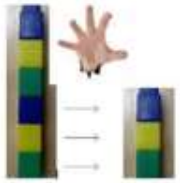
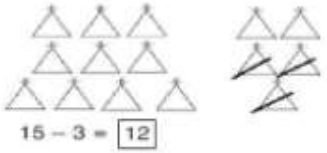
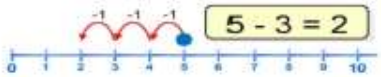
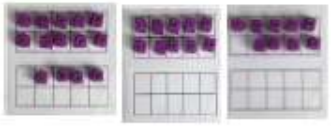
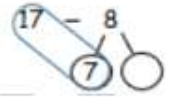
- Sum
- total
- parts and wholes
- Plus
- Add
- altogether
- more
- 'is equal to'
  'is the same as'

	9+3=12 This example shows how bead strings can be used to demonstrate the same method. 		
Use number bonds to 10 to make number bonds to 20 Addition Year 1	Use 10s frames and coloured counters (1 frame for the number bond to 10 and 2 frames for the number bond to 20) and Numicon. 	Colour in dots (2 different colours to make the bonds) on 10s frames (1 frame for the number bond to 10 and 2 frames for the number bond to 20). 	7+3=10 so 17+3=20 or 13+7=20

Maths | Modelling - Year 1: Subtraction

Key Vocabulary

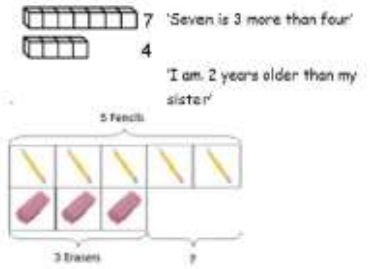
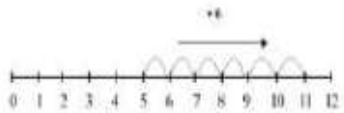
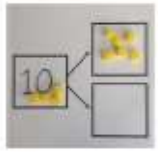
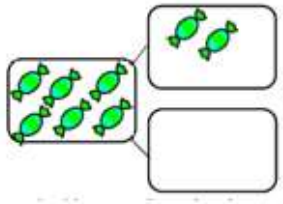
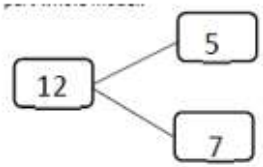
- take away
- less than
- the difference
- subtract
- minus
- fewer
- decrease

Year 1 Subtraction Use dienes (tens and ones/units) for number in preparation for year 2			
Objective/Strategy	**Concrete**	**Pictorial**	**Abstract**
Taking away 1-digit then 2-digit numbers up to 20 (start by not crossing 10, then crossing 10) Subtraction Year 1	Use physical objects, counters, cubes, bead strings, Numicon, etc. to show how objects can be taken away. 4—2 = 2   6 -3=3  20 – 7 = ___	Cross out drawn objects to show what has been taken away, count back on a number line, bar model.  15 – 3 = 12  5 - 3 = 2	7—4 = 3 16—9 = 7 Put 13 in your head, count back 4. What number are you at?
Make 10 when counting back to cross over 10 Subtraction Year 1	14—5= Make 14 on the ten frame with counters. Take 4 away to make ten, then take one more away so that you have taken 5. 	Use a number line, jump back 3 first, then another 4. Use ten as the stopping point. Use part-whole model.  	16—8 How many do we take off first to get to 10? How many left to take off?
Find the difference (relate to addition, counting on as well as counting back)	Compare objects and amounts Lay objects to represent bar model.	Count on using a number line to find the difference.	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?

Maths | Modelling - Year 1: Subtraction

Key Vocabulary

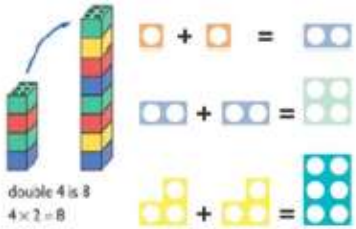
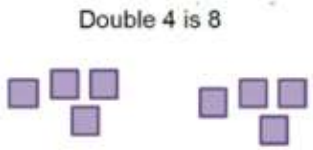
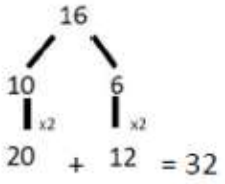
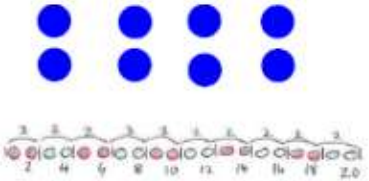
- take away
- less than
- the difference
- subtract
- minus
- fewer
- decrease

Subtraction Year 1			
Represent and use number bonds and related subtraction facts within 20. Include subtracting zero. Part-whole model Subtraction Year 1	Link to addition. 12+1=13 13-1=12 Use 10s frames and 2 different coloured counters to model inverse.  If 10 is the whole and 6 is one of the parts, what is the other part? 10 – 6 = 4	Use part-whole model (dienes drawn), bar model and draw dots in 10s frames. 	Move to using numbers within the part-whole model. Include missing number problems: 12 - ? = 5 7 = 12 - ? 

Maths | Modelling - Year 1: Multiplication

Key Vocabulary

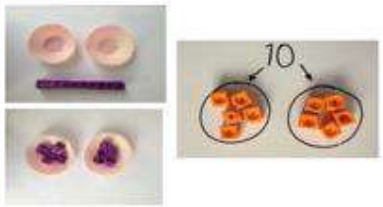
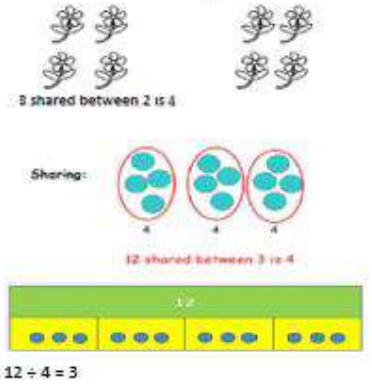
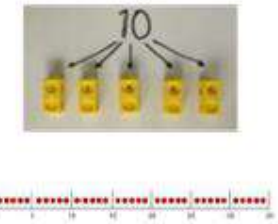
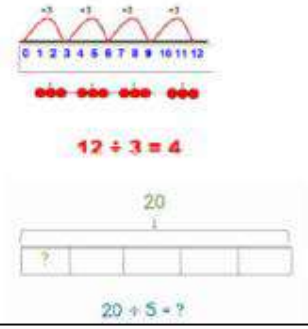
- Double
- Times
- multiplied by
- the product of
- groups of
- lots of
- equal groups

Year 1 Multiplication			
Objective/Strategy	**Concrete**	**Pictorial**	**Abstract**
Doubling Multiplication Year 1	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling. 	Draw pictures to show how to double numbers. 	Partition a number and then double each part before recombining it back together. 
Counting in multiples (2s, 5s, 10s) Multiplication Year 1	Count the groups of 2, 5 or 10 using bead strings, number lines, 100 square, Numicon, looking at images of groups. Children could use their fingers as they are counting. 	Children draw representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Maths | Modelling - Year 1: Division

Key Vocabulary

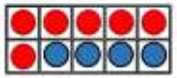
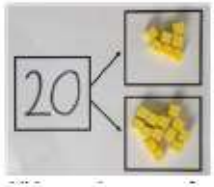
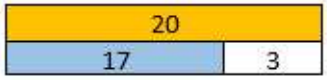
- Double
- Times
- multiplied by
- the product of
- groups of
- lots of
- equal groups

Year 1 Division			
Objective/Strategy	**Concrete**	**Pictorial**	**Abstract**
Division as sharing (children do not need to be familiar with the symbol yet) Division Year 1	I have 10 cubes, can you share them equally in 2 groups? 	Children use pictures and bar models to share quantities. E.g draw pictures of sharing 10 muffins between 2 plates. 	12 shared between 3 is 4. (12 ÷ 3 = 4)
Introduce division as grouping (children do not need to be familiar with the symbol yet) Division Year 1	Divide quantities into equal groups e.g. 20 counters in total how many equal groups of 2 can you make? Use cubes, counters, objects or place value counters to aid understanding. 	If you have a total of 12, how many equal groups of 3 will you have? Use a number line or a bar model. 	Divide 28 into 4s. How many equal groups do you get? (28 ÷ 4 =)

Maths | Modelling - Year 2: Addition

Key Vocabulary

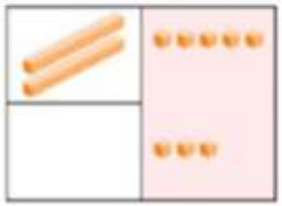
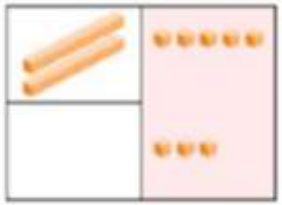
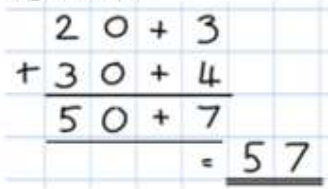
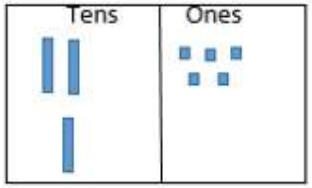
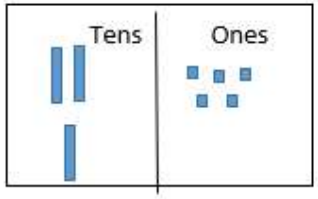
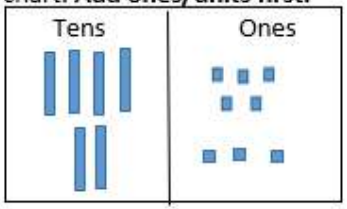
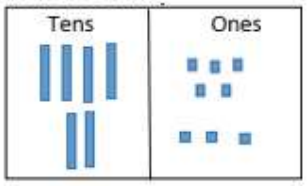
- Sum
- Total
- parts and wholes
- Plus
- Add
- Altogether
- More
- 'is equal to'
- 'is the same as'

Year 2 Addition up to 100 and starting to cross over 100			
Objective/Strategy	**Concrete**	**Pictorial**	**Abstract**
Adding multiples of ten to make 100 and numbers up to 100. Addition Year 2	Model creating number bonds using a 10s frame to represent 100 and 2 different coloured counters, dienes or Numicon 10s.  (Each counter represents 10 in this example) 60 + 40 = 100 	Drawing of 10s rods, 10s numberline or 10s place value counters on a 10s frame to represent 100.  60 + 40 = 100	20 + 30 = 50 70 = 50 + 20 40 + □ = 60 Recite number bonds to 100
Use known addition number facts to make 20 to explore **subtraction** facts (to make 20 and numbers up to 20) links to Year 2 subtraction Addition Year 2	Children explore ways of making numbers within 20 using part-whole model, counters or cubes and 2 tens frames with 2 different colour counters. 	Use bar model, number lines, part-whole model with numbers, drawings of 10s frames with dots.   	Explore commutativity of addition and make list. E.g. 12+3=15 15=12+3 3+12=15 15=3+12 Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations. E.g. 12+3=15 So 15-3=12 and 15-12=3

Maths | Modelling - Year 2: Addition

Key Vocabulary

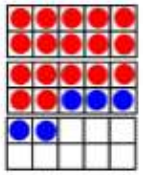
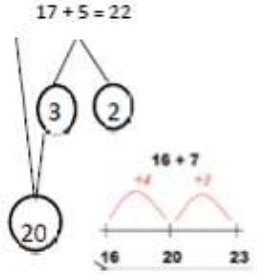
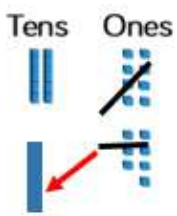
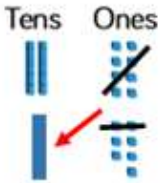
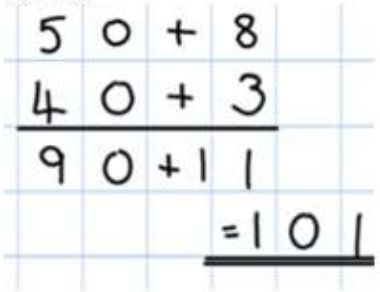
- Sum
- Total
- parts and wholes
- Plus
- Add
- Altogether
- More
- 'is equal to'
- 'is the same as'

Add a two-digit number and ones (not bridging 10s, so no exchanging) Addition Year 2 See also Empty Number line method as an alternative (below)	Use dienes and place value chart. **Add ones/units first.** 	Use part-whole model or number track to model. When bridging 10s use a number line, draw dienes in place value chart. Add ones/units first. 	Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values. 
Add a 2-digit number and tens Addition Year 2	25 + 10 = 35 Explore that the ones digit does not change. Use dienes in a place value chart. **Add ones/units first.** 	Draw dienes in the place value chart to help to lead into column addition. Look at ones/units column first to see if there's anything to add.  25 + 10 = 35	27 + 10 = 37 27 + 20 = 47 27 + ☐ = 57
Add two, 2-digit numbers (no exchanging) Addition Year 2	45 + 23 = Use dienes in a place value chart. **Add ones/units first.** 	Draw dienes in the place value chart to help to lead into column addition. Add ones then add tens. 	27 + 31 = 58 31 + ☐ = 58

Maths | Modelling - Year 2: Addition

Key Vocabulary

- Sum
- Total
- parts and wholes
- Plus
- Add
- Altogether
- More
- 'is equal to'
- 'is the same as'

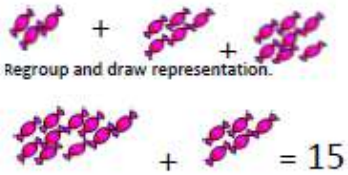
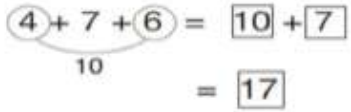
	Add the ones then add the tens.		
Regrouping to make 10 *This is an essential skill for column addition* Addition Year 2	Start with the bigger number and use the smaller number to make 10. Use ten frames.  17 + 5 = 22 17 + 3 = 20 20 + 2 = 22 Explore the pattern 17 + 5 = 22 27 + 5 = 32	Regroup or partition the smaller number using the part-whole model to make 10, a number line, 10s frames. 	Regroup to make other multiples of 20 + E.g. 33+9= 33+7+2=
Add two numbers **exchanging** 1s for 10s Addition Year 2	Model using dienes on a place value chart, exchange ten 1s for a tens rod and move that with the 10s. **Add ones/units first.** 28+7= 	Model drawing dienes on a place value chart, exchange ten 1s for a tens rod and move that with the 10s by crossing out the ten 1s and drawing the exchanged 10 under the tens column. 28+7= 	Use expanded column addition format. 

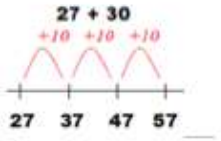
Maths | Modelling - Year 2: Addition

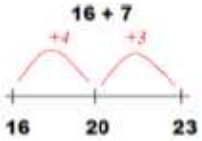
Key Vocabulary

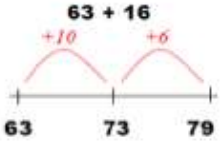
- Sum
- Total
- parts and wholes
- Plus
- Add
- Altogether
- More
- 'is equal to'
- 'is the same as'

<table>
<tr>
<td>Add three 1-digit Numbers

Addition Year 2</td>
<td>Combine to make 10 first if possible, or bridge 10 then add third digit. Use 3 different coloured counters, cubes.
  </td>
<td colspan="2"> 
</td>
<td>Combine the two numbers that make/ bridge ten then add on the third.
 
</td>
</tr>
<tr>
<td>Add with an empty number line

Addition Year 2</td>
<td colspan="2">Add multiples of 10
 

Add 1-digit numbers
 

Add pairs of 2-digit numbers
 
</td>
<td>For concrete support use a 100 square alongside or dienes.

For the pictorial stage, add on ready drawn number lines.</td>
<td>Independently use the empty number line method (apparatus can still be used alongside).</td>
</tr>
</table>

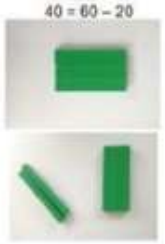
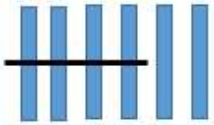
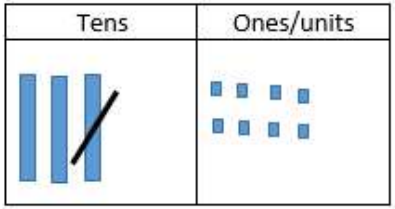
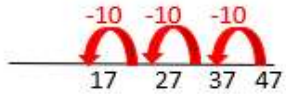
Maths | Modelling - Year 2: Subtraction

Key Vocabulary

- take away
- less than
- the difference
- subtract
- Minus
- Fewer
- Decrease

Year 2 Subtraction

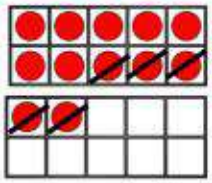
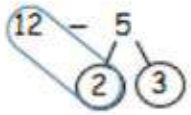
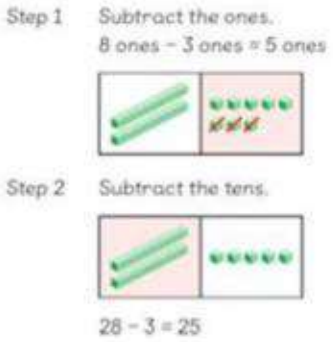
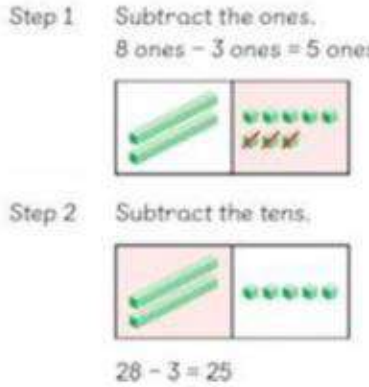
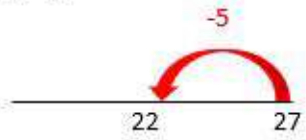
Subtraction number facts included in Year 2 Addition

Objective/Strategy	Concrete	Pictorial	Abstract
Subtracting multiples of ten to make 100 and numbers up to 100 Subtraction Year 2	Model using a 10s frame to represent 100 and 2 different coloured counters to create number bonds, dienes or Numicon 10s. 	Drawing of 10s rods and cross out 10s, 10s numberline or 10s place value counters on a 10s frame to represent 100. 	30 - 20= 10 70 = 100 - 30 90 - ☐ = 60 Recite subtraction number bonds to 100
Subtract multiples of 10 from numbers to 100 Subtraction Year 2 See also Empty Number line method as an alternative (below)	Use a place value chart with dienes (start to lead into column subtraction). **Subtract ones/units first.** 38 – 10 = 	Use a place value chart and draw dienes (start to lead into column subtraction). 38 – 10 = 	Subtract multiples of ten 47 – 30 = 
Make 10 when counting back to cross over 10 Subtraction Year 2	Use 10s frames to subtract back to the 10 by partitioning the second number. 12 -5=	Use number line and part-whole model to subtract back to the 10 by partitioning the second number.	Use the strategy mentally e.g. 23 – 5 = so 23 – 3 = 20, then there's 2 left to subtract (because 3 + 2 = 5) so 20 – 2 = 18

Maths | Modelling - Year 2: Subtraction

Key Vocabulary

- take away
- less than
- the difference
- subtract
- Minus
- Fewer
- Decrease

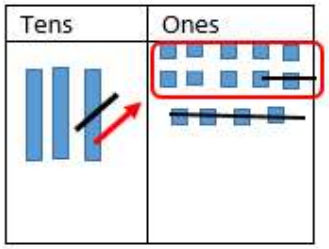
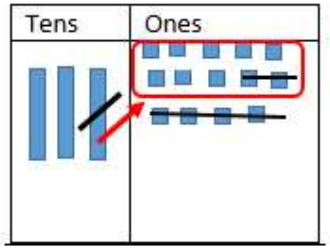
	 12 – 2 = 10 10 – 3 = 7	13 – 5 =  12 – 5 = 	
Subtract a 1-digit number from a 2-digit (no exchanging) Subtraction Year 2	Use dienes, subtract by moving ones away. **Subtract ones/units first.** 	Use drawings of dienes in a place value chart and cross out to subtract. 	27 – 5 = 

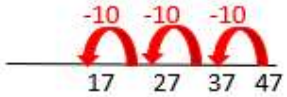
Maths | Modelling - Year 2: Subtraction

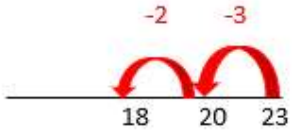
Key Vocabulary

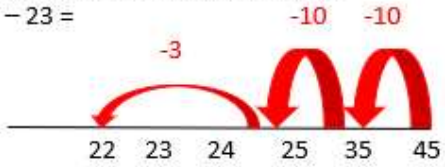
- take away
- less than
- the difference
- subtract
- Minus
- Fewer
- Decrease

<table>
<tr>
<td>Subtract two 2-digit numbers (exchanging 10) Included in Year 2

Subtraction Year 2</td>
<td>Use dienes, exchange a 10 for ten 1s/units. Subtract ones/units first.
34 – 16 =
 
</td>
<td>Use drawings of dienes in a place value chart and to exchange 10, cross out 10s rod and draw ten ones then cross out amount to subtract.
 
</td>
<td></td>
</tr>
<tr>
<td>Subtract with an empty number line

Subtraction Year 2</td>
<td>Subtract multiples of ten
47 – 30 =
 

Subtract 1-digit numbers
23 – 5 =
 

Subtract pairs of 2-digit numbers
45 – 23 =
 
</td>
<td>For concrete support use a 100 square alongside or dienes.

For the pictorial stage, subtract on ready drawn number lines.</td>
<td>Independently use the empty number line method (apparatus can still be used alongside).</td>
</tr>
</table>

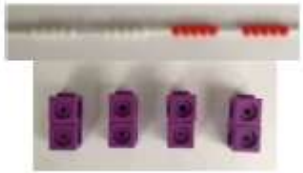
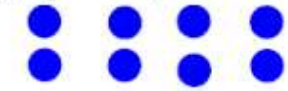
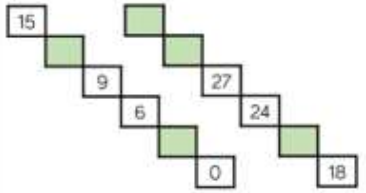
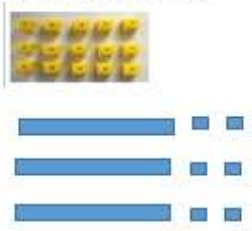
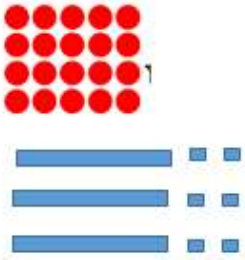
Maths | Modelling - Year 2: Multiplication

Key Vocabulary

- Double
- Times
- multiplied by
- the product of
- groups of
- lots of
- equal groups

Year 2 Multiplication

Children should be able to recall and use the multiplication and division facts for 2, 5 and 10 x table
Links between multiplication and division in the division section

Objective/Strategy	Concrete	Pictorial	Abstract
Counting in multiples of 3 (recap 2, 5 and 10) Multiplication Year 2	Count the groups of 2, 5, 10 and 3s using bead strings, number lines, 100 square, Numicon, looking at images of groups. 	Draw number lines counting in groups for hops.  	Count in multiples of a number aloud. Write sequences with multiples of numbers (fill in missing numbers from pattern). 0, 3, 6, 9, 12, 15 
Using arrays to solve multiplication calculations Multiplication Year 2	Use objects including dienes laid out in arrays to find the answers to 3 x 5, 3 x 12 etc. 	Draw representations of arrays to solve multiplication calculations. 	Write different calculations for an image of an array including + and x.  5 + 5 + 5 + 5 = 20 4 + 4 + 4 + 4 + 4 = 20 4 x 5 = 20 5 x 4 = 20

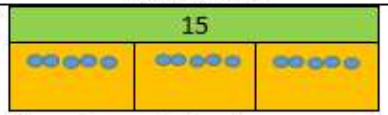
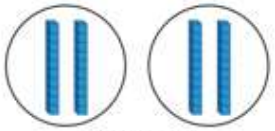
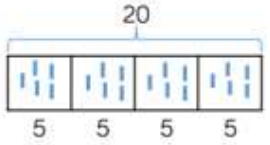
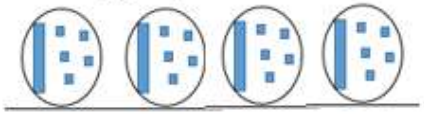
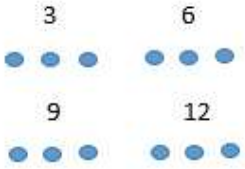
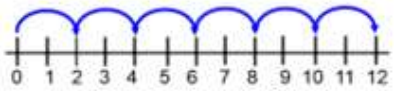
Maths | Modelling - Year 2: Division

Key Vocabulary

- Share
- group
- Divide
- divided by
- half

Year 2 Division

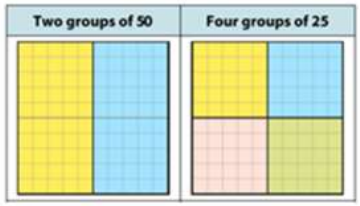
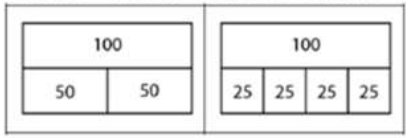
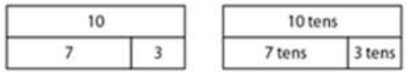
Children should be able to recall and use the multiplication and division facts for 2, 5 and 10 x table

Objective/Strategy	Concrete	Pictorial	Abstract
Division as sharing (with ÷ symbol) Division Year 2	 Use a bar model or draw groups to solve division calculations with counters, cubes or dienes.  40 ÷ 2 = 20	Draw a bar model/draw pictures to solve division calculations.  60 ÷ 4 =  Children will need to exchange 2 tens for 20 ones/units so they can put 1 ten and 5 ones in each group.	How many different ways can you divide/share equally 20? 20 ÷ 1 = 20 20 ÷ 20 = 1 20 ÷ 2 = 10 20 ÷ 10 = 2 20 ÷ 4 = 5 20 ÷ 5 = 4
Division as grouping (with ÷ symbol) Division Year 2	Divide quantities into equal groups e.g. 12÷3= ; get 12 counters/cubes divide them into 3s. How many groups are there? 	Use bar modelling and a number line to aid solving division problems by grouping. 	How many groups of 4 in 24? 24 ÷ 4 = ? 24 ÷ ? = 4

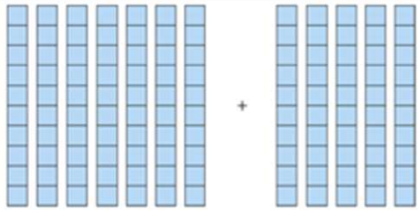
Maths | Modelling - Year 3: Addition and Subtraction

Year 3 – addition and subtraction

Application of Number Facts

	Concrete and Pictorial Representations	Abstract
Number facts to 100 (These are explored both as additive and multiplicative equations) **Stem Sentences**	  25 stickers 25 stickers 25 stickers 25 stickers 	100 = 50 + 50 100 = 2 × 50 100 = 50 × 2 100 ÷ 2 = 50 100 ÷ 50 = 2 100 = 25 + 25 + 25 + 25 100 = 4 × 25 100 = 25 × 4 100 ÷ 4 = 25 100 ÷ 25 = 4 100 = 25 + ☐ + 25 + 25 100 = 50 + ☐ 100 – 25 = ☐ 100 – 50 = ☐ 100 = 4 × ☐ 2 × ☐ = 100 100 ÷ 4 = ☐ ☐ = 100 ÷ 2 100 – 20 – 20 = ☐ ☐ = 100 – 10 – 10 – 10
Known addition number facts with single digit numbers can be used to calculate complements to 100 and add and subtract across 100	 	7 + 3 = 10 10 – 3 = 7 70 + 30 = 100 100 – 30 = 70

Maths | Modelling - Year 3: Addition and Subtraction

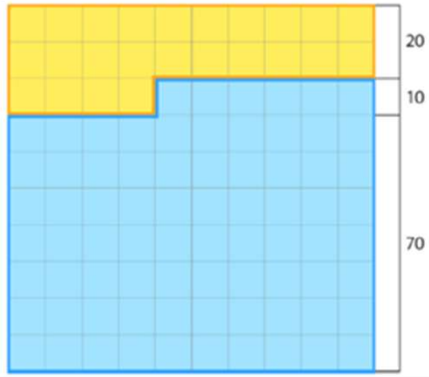


- *'I know that seven plus five is equal to twelve.'*
- *'So seven tens plus five tens is equal to twelve tens.'*
- *'Seventy plus fifty is equal to one hundred and twenty.'*

7	+ 5	= 12
7 tens	+ 5 tens	= 12 tens
70	+ 50	= 120

Partitioning

Addition of complements to 100 can be done by partitioning both addends (the numbers being added together) into tens and ones

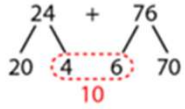


24 + 76

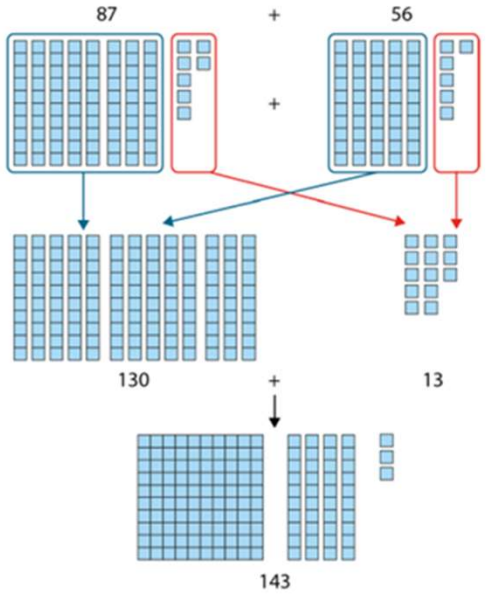
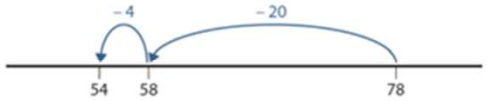
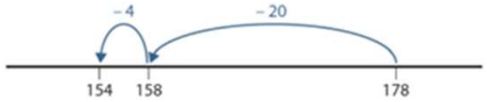
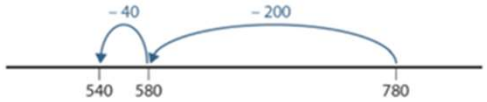
20 + 70 = 90

4 + 6 = 10

90 + 10 = 100



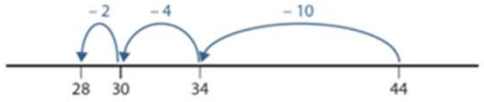
Maths | Modelling - Year 3: Addition and Subtraction

Partitioning can be used to add both two-digit and three-digit numbers	Dienes:  	Jottings: 
Partitioning the subtrahend (the number being taken away) can be used when subtracting. This method can be used either without bridging or with bridging.	Partitioning the subtrahend (without bridging): 78 – 24  178 – 24  780 – 240  	

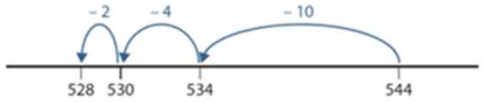
Maths | Modelling - Year 3: Addition and Subtraction

Partitioning the subtrahend (with bridging):

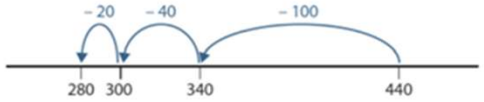
44 – 16



544 – 16



440 – 160

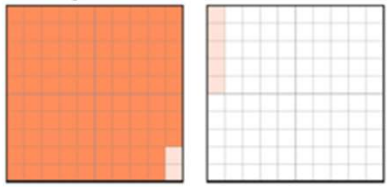


Making 100

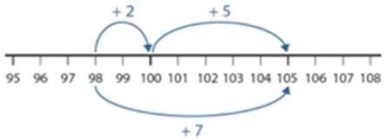
Bridging 100 (adding or subtracting across 100) can be done by first making 100

98 + 7

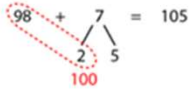
Hundred grids:



Number line:



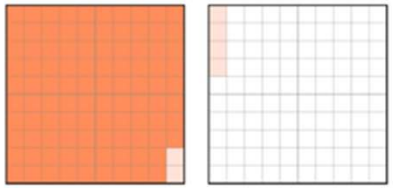
Jotting and equations:



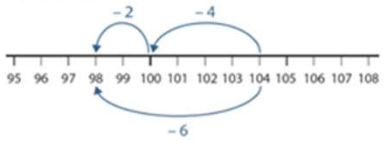
98 + 7 = 98 + 2 + 5
= 100 + 5
= 105

104 – 6

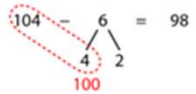
Hundred grids:



Number line:



Jotting and equations:



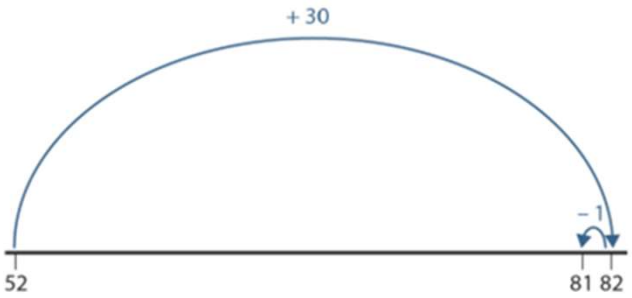
104 – 6 = 104 – 4 – 2
= 100 – 2
= 98

Maths | Modelling - Year 3: Addition and Subtraction

Adjusting

Adjusting is a more efficient addition strategy than partitioning when one of the numbers involved is close to a multiple of 10 or 100 (e.g. 49 is close to 50).

In the example given, 30 is added rather than 29 as it is a simpler calculation. 1 is then subtracted to adjust for the extra 1 that was added.



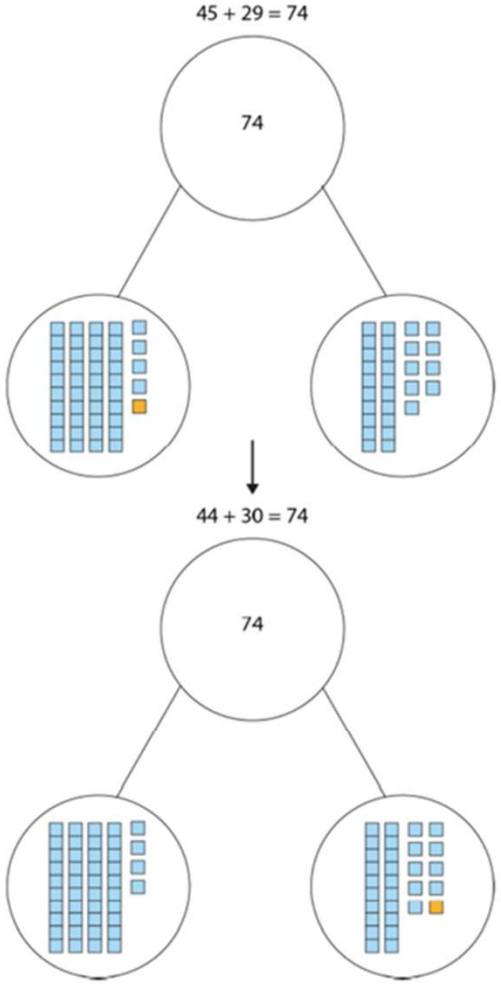
$$\begin{aligned} 52 + 29 &= 52 + 30 - 1 \\ &= 82 - 1 \\ &= 81 \end{aligned}$$

Maths | Modelling - Year 3: Addition and Subtraction

Redistributing

In the redistribution strategy an addition calculation is made simpler by increasing one addend and decreasing the other by the same amount.

There are similarities between the redistributing strategy and the adjusting strategy. However, with redistribution, the total remains the same at all times whereas with adjusting the total amount is increased to simplify the calculation and then decreased again.



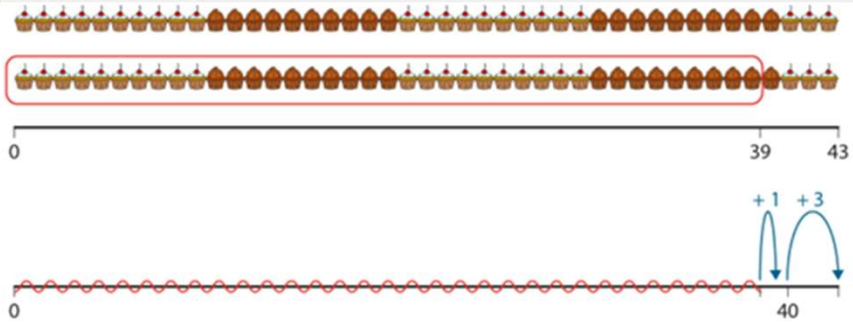
45 + 29 = 44 + 30 = 74

Maths | Modelling - Year 3: Addition and Subtraction

Finding the difference (adding on)

In this strategy, start with the subtrahend and add on to reach the minuend. The amount needed to be added will be the difference and the answer to the calculation.

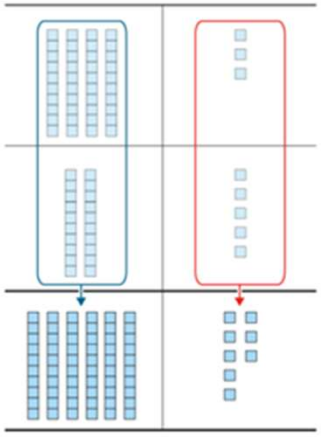
This strategy is particularly useful when the minuend and subtrahend are close together (e.g. 43 – 39)



$43 - 39 = 4$

Column Addition

Column addition is the formal written method for addition taught and used throughout KS2 and beyond for times when an efficient mental method is either not known or cannot be used to a suitable degree of accuracy.

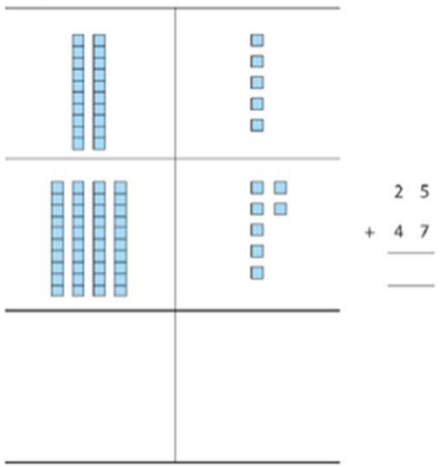


$$\begin{array}{r} 43 \\ +\ 25 \\ \hline 68 \end{array}$$

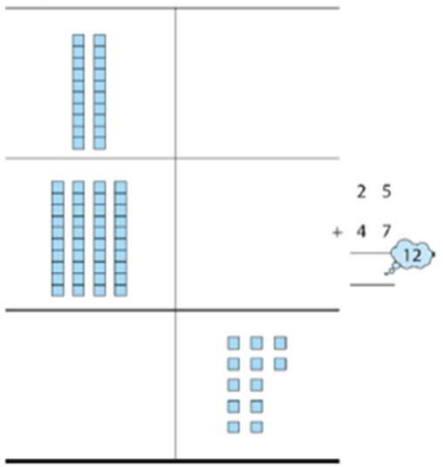
Maths | Modelling - Year 3: Addition and Subtraction

When the total of any column is 10 or greater, we must regroup. In the example shown, this involves exchanging 10 ones within the number 12 for 1 ten. This leaves 2 in the ones column and 1 ten below the tens column to be added when the tens are added.

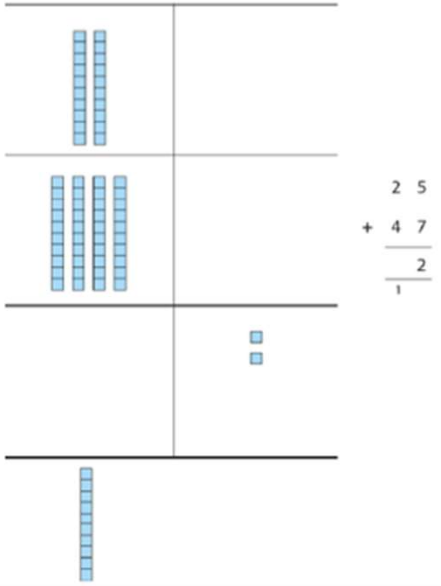
Step 1



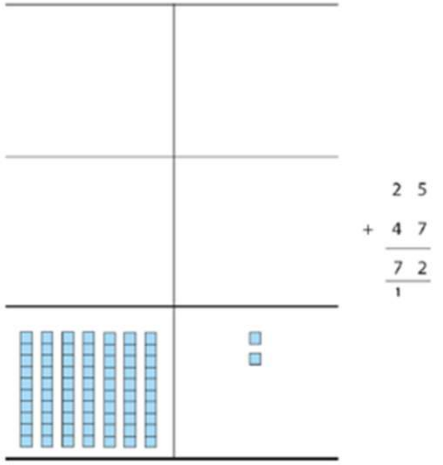
Step 2



Step 3



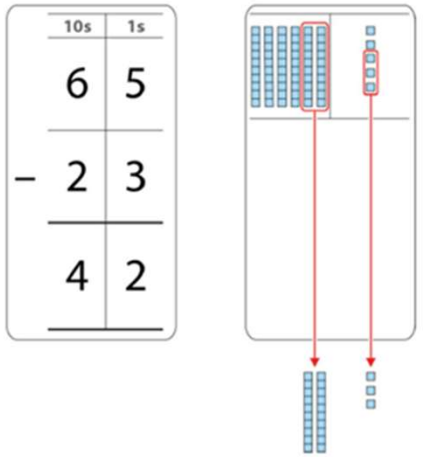
Step 4



Maths | Modelling - Year 3: Addition and Subtraction

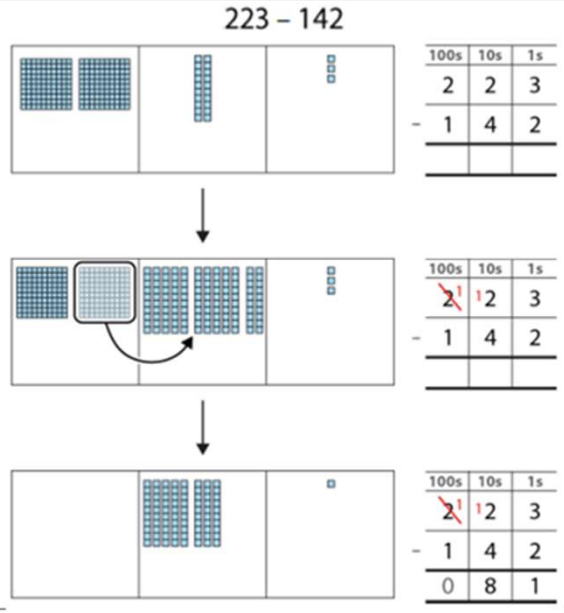
Column Subtraction

Similar to column addition, columns subtraction is the formal written method for subtraction taught and used throughout KS2 and beyond for times when an efficient mental method is either not known or cannot be used to a suitable degree of accuracy.



When the subtrahend (the number on the second row) in any column is greater than the minuend above it (the number at the top), we must regroup.

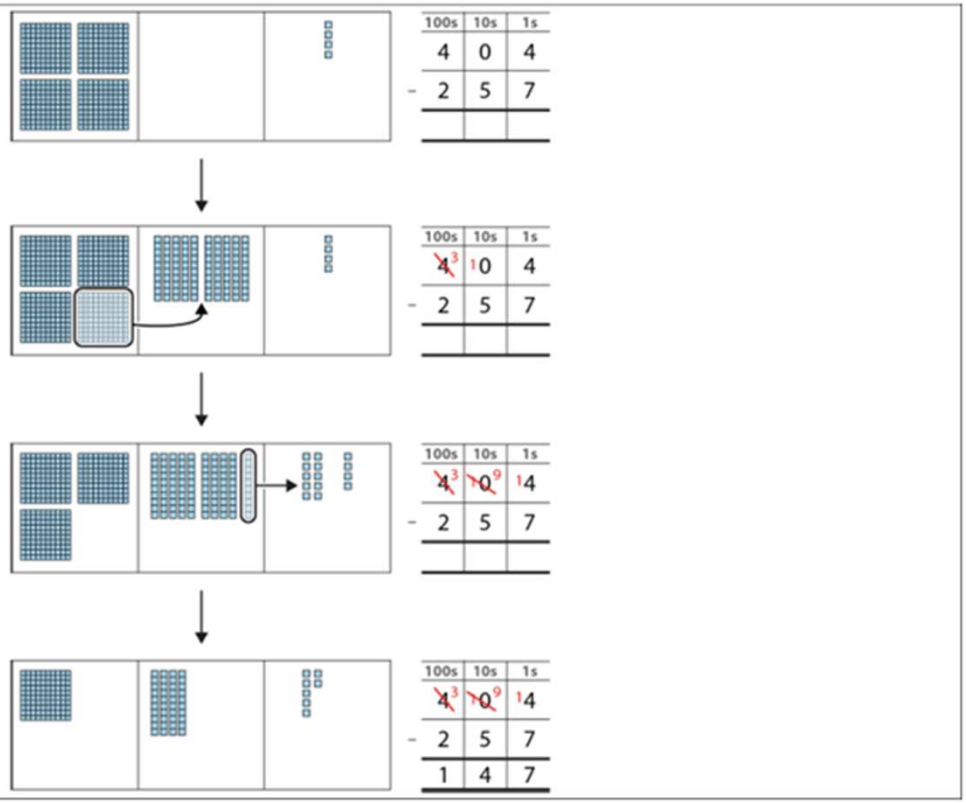
In the example shown (where 4 tens cannot be taken away from 2 tens), this involves exchanging 1 of the hundreds for 10 tens leaving 1 hundred remaining in the hundreds column and combining the exchanged 10 tens with the existing 2 tens to give 12 tens in the tens column. This allows 4 tens to be taken away from the 12 tens.



Maths | Modelling - Year 3: Addition and Subtraction

Regrouping can sometimes require working through a column with zero because the zero shows there is nothing to be exchanged.

In this situation, as shown in the example, regrouping can be done by exchanging from the next column to the left (the hundreds in this case). The regrouping must first be done into the column with zero (so exchanging 1 hundred into ten tens) which can then lead to regrouping into the column where the initial subtraction wasn't possible (so exchanging 1 of the previously exchanged tens into 10 ones).



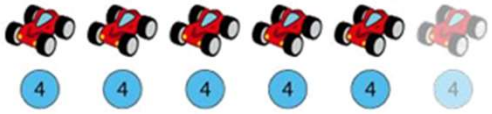
Maths | Modelling - Year 3: Multiplication and Division

Year 3 – multiplication and division

Within Year 3, the children continue to develop their times table knowledge by recalling the 5x and 2x tables learnt in KS1 and learning their times table number facts for the 4x and 8x tables. The 2x, 4x and 8x tables are taught in this sequence to reinforce the doubling relationship between them. Once the 2x, 4x and 8x tables are secure, the children then learn the 3x, 6x and 9x tables and the relationship between them.

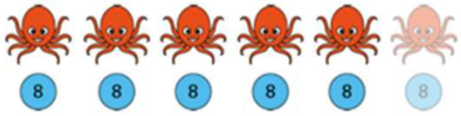
While a formal written method for multiplication and division is not taught in Year 3, the acquisition of times table knowledge is essential for the children to be ready to learn these in Year 4.

Number Facts

	Concrete and Pictorial Representations	Abstract
Times table number facts: 5x and 2x table (This is a recap of KS1 learning)		
Times table number facts: 2x, 4x and 8x table and the relationship between them **STEM SENTENCES**	 	

Number of cars	Total number of wheels
0	0
1	4
2	8
3	12
4	16
5	20
6	**24**

0 × 4 = 0	4 × 0 = 0
1 × 4 = 4	4 × 1 = 4
2 × 4 = 8	4 × 2 = 8
3 × 4 = 12	4 × 3 = 12
4 × 4 = 16	4 × 4 = 16
5 × 4 = 20	4 × 5 = 20
6 × 4 = 24	**4 × 6 = 24**

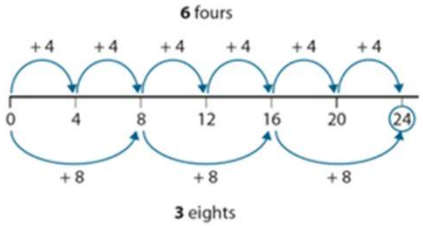


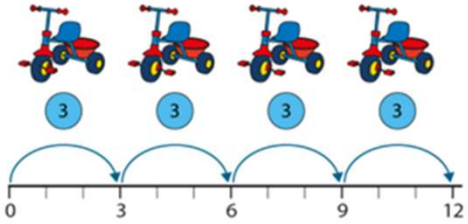
Number of octopuses	Total number of tentacles
0	0
1	8
2	16
3	24
4	32
5	40
6	**48**

0 × 8 = 0	8 × 0 = 0
1 × 8 = 8	8 × 1 = 8
2 × 8 = 16	8 × 2 = 16
3 × 8 = 24	8 × 3 = 24
4 × 8 = 32	8 × 4 = 32
5 × 8 = 40	8 × 5 = 40
6 × 8 = 48	**8 × 6 = 48**

Maths | Modelling - Year 3: Multiplication and Division

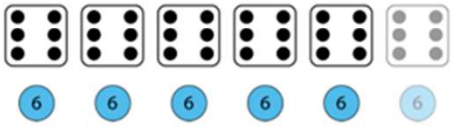
Times table number facts: 3x, 6x and 9x table and the relationship between them





Number of tricycles	Total number of

0 × 3 = 0	3 × 0 = 0
1 × 3 = 3	3 × 1 = 3

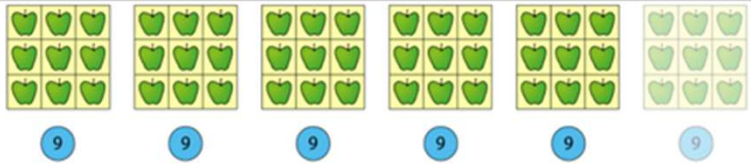


Number of six-value dice	Total number of dots
0	0
1	6
2	12
3	18
4	24
5	30
6	**36**

0 × 6 = 0	0 × 6 = 0
1 × 6 = 6	1 × 6 = 6
2 × 6 = 12	2 × 6 = 12
3 × 6 = 18	3 × 6 = 18
4 × 6 = 24	4 × 6 = 24
5 × 6 = 30	5 × 6 = 30
6 × 6 = 36	**6 × 6 = 36**

Number	**0**	1	2	**3**	4	5	**6**	7	8	**9**	10	11	**12**	13	14	**15**	16	17	**18**	19	20	**21**	22	23	**24**
Counting in 3s	✓			✓			✓			✓			✓			✓			✓			✓			✓
Counting in 6s	✓						✓						✓						✓						✓

Maths | Modelling - Year 3: Multiplication and Division



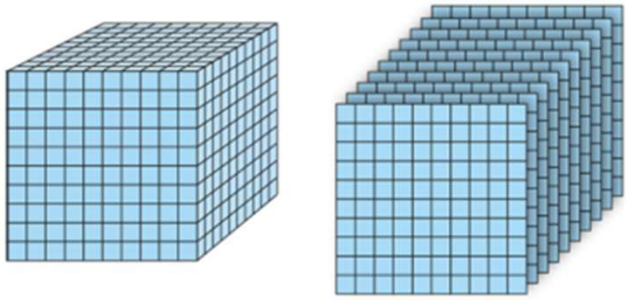
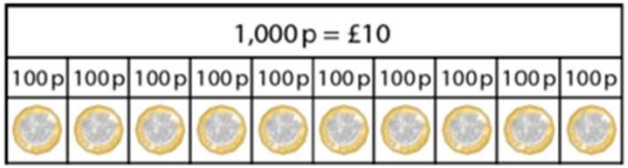
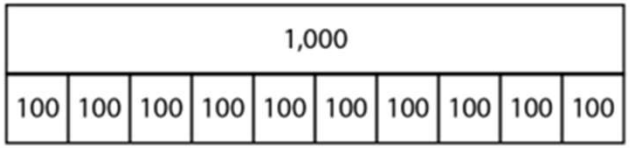
Number of boxes of 9 apples	Total number of apples
0	0
1	9
2	18
3	27
4	36
5	45
6	**54**

$0 \times 9 = 0$	$9 \times 0 = 0$
$1 \times 9 = 9$	$9 \times 1 = 9$
$2 \times 9 = 18$	$9 \times 2 = 18$
$3 \times 9 = 27$	$9 \times 3 = 27$
$4 \times 9 = 36$	$9 \times 4 = 36$
$5 \times 9 = 45$	$9 \times 5 = 45$
$\mathbf{6 \times 9 = 54}$	$\mathbf{9 \times 6 = 54}$

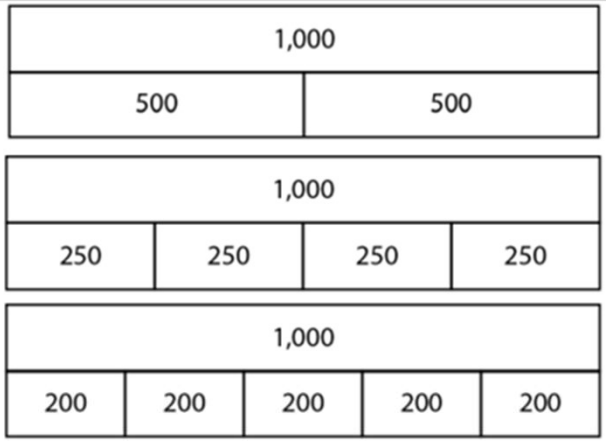
Maths | Modelling - Year 4: Addition and Subtraction

Year 4 – addition and subtraction

Application of Number Facts

	Concrete and Pictorial Representations	Abstract
Number facts to 1,000 (These are explored both as additive and multiplicative equations and applied within the range of strategies listed below)	Representing ten hundreds in 1,000: • Tens frame and 100 place-value counters  • Dienes  • Coins  	• Additive and multiplicative equations 1,000 = 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 1,000 = 10 × 100 1,000 = 100 × 10 1,000 ÷ 100 = 10 1,000 ÷ 10 = 100

Maths | Modelling - Year 4: Addition and Subtraction

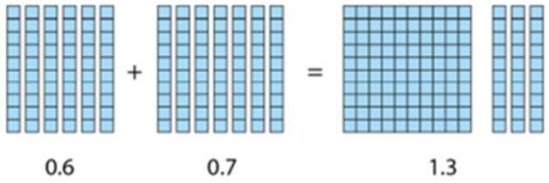


Mental Strategies with 4-digit numbers and decimals

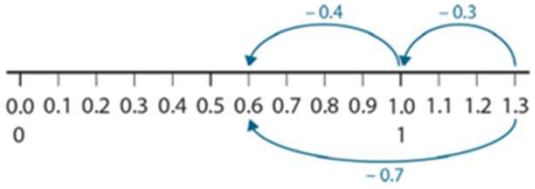
Each of the mental strategies taught in Year 3 are applied and explored within numbers in the thousands. This gives a chance for the children to be reminded of those strategies and gain familiarity in using them with increasing confidence while working with 4-digit numbers. These are further developed and extended once the children have learnt about decimals (tenths, hundredths and thousands). Examples of these strategies being used for decimals can be seen below.

See the Year 3 section above for an explanation and example of each mental strategy covered.

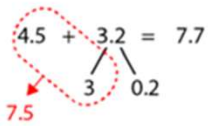
Dienes – example addition calculation:



Number line – example subtraction calculation:



Applying known strategies – partitioning numbers with tenths:



Maths | Modelling - Year 4: Multiplication and Division

Column Addition and subtraction

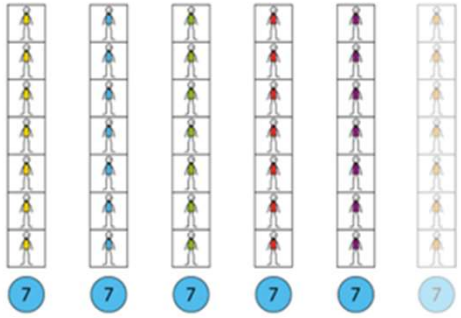
The column addition and subtraction algorithms taught in Y3 are extended and built upon in Y4 to include addition and subtraction of 4-digit numbers and decimals. For an explanation of these methods and how they are introduced, see the Y3 section above.		$\begin{array}{r} 13.2 \\ +\ 5.7 \\ \hline \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 36.5 \\ -\ 2.3 \\ \hline \\ \hline \end{array}$

Year 4 – multiplication and division

Initially, the times table facts taught in Y3 are recapped to ensure these are secure before moving on to learn the 7x, 11x and 12x tables. This prepares them for a range of mental and written strategies for multiplication and division as well as for the statutory Multiplication Tables Check (a test given to all children in Year 4 to assess fluency of times tables recall).

Children in Year 4 are also taught a formal written method for multiplication and division: short multiplication and short division.

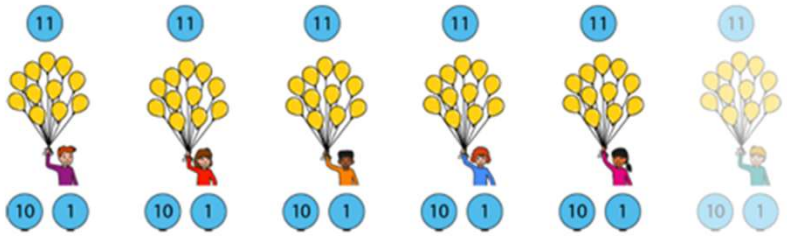
Number Facts

	Concrete and Pictorial Representations	Abstract
Times table number facts: 7x, 11x and 12x tables		

Number of netball teams	Total number of players
0	0
1	7
2	14
3	21
4	28
5	35
6	**42**

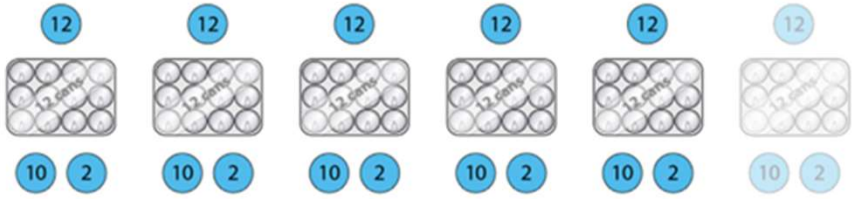
$0 \times 7 = 0$	$7 \times 0 = 0$
$1 \times 7 = 7$	$7 \times 1 = 7$
$2 \times 7 = 14$	$7 \times 2 = 14$
$3 \times 7 = 21$	$7 \times 3 = 21$
$4 \times 7 = 28$	$7 \times 4 = 28$
$5 \times 7 = 35$	$7 \times 5 = 35$
$\mathbf{6 \times 7 = 42}$	$\mathbf{7 \times 6 = 42}$

Maths | Modelling - Year 4: Multiplication and Division



Number of bunches of balloons	× 10	× 1	Total number of balloons (× 11)
0	0	0	0
1	10	1	11
2	20	2	22
3	30	3	33
4	40	4	44
5	50	5	55
6	**60**	**6**	**66**

0 × 11 = 0	11 × 0 = 0
1 × 11 = 11	11 × 1 = 11
2 × 11 = 22	11 × 2 = 22
3 × 11 = 33	11 × 3 = 33
4 × 11 = 44	11 × 4 = 44
5 × 11 = 55	11 × 5 = 55
6 × 11 = 66	**11 × 6 = 66**



0 × 12 = 0	12 × 0 = 0
1 × 12 = 12	12 × 1 = 12
2 × 12 = 24	12 × 2 = 24
3 × 12 = 36	12 × 3 = 36
4 × 12 = 48	12 × 4 = 48
5 × 12 = 60	12 × 5 = 60
6 × 12 = 72	**12 × 6 = 72**

Maths | Modelling - Year 4: Multiplication and Division

Number of packs of cans	× 10	× 2	Total number of cans (× 12)
0	0	0	0
1	10	2	12
2	20	4	24
3	30	6	36
4	40	8	48
5	50	10	60
6	**60**	**12**	**72**

Multiplying and dividing by 10 and 100

	Concrete and Pictorial Representations	Abstract

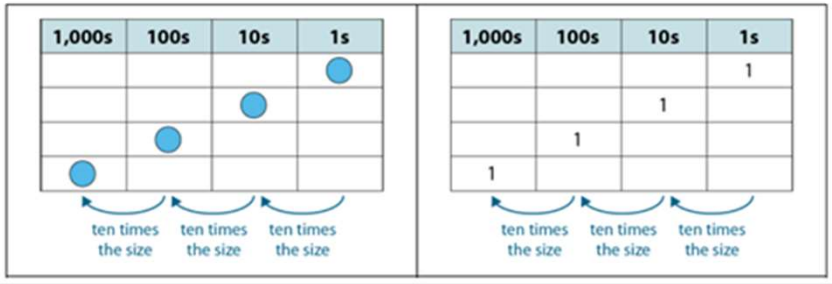
The mental strategy for multiplying and dividing by 10 and 100 involves recognising the patterns within place value columns.

For example, when a number is multiplied by 10, all of the digits move one place to the left (the number in the 1s moving to the 10s). This means that all of the digits will stay in the same order but will have a place holder in the 1s column).

Gattegno chart:

	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
	100	200	300	400	500	600	700	800	900
× 10	10	20	30	40	50	60	70	80	90
	1	2	3	4	5	6	7	8	9

÷ 10



Maths | Modelling - Year 4: Multiplication and Division

Step 1 – move each of the digits one place to the left

1,000s	100s	10s	1s
		1	2
	1	2	

↓ × 10 *Think of '12' and make it ten times the size.*

ten times the size, ten times the size, ten times the size

Step 2 – write a '0' in the ones place

1,000s	100s	10s	1s
		1	2
	1	2	0

↓ × 10 *Think of '12' and make it ten times the size.*

ten times the size, ten times the size, ten times the size

Step 1 – move each of the digits two places to the left

1,000s	100s	10s	1s
		1	5
1	5		

↓ × 100 *Think of '15' and make it 100 times the size.*

100 times the size, 100 times the size

Step 2 – introduce zeros in the tens and ones places

1,000s	100s	10s	1s
		1	5
1	5	0	0

↓ × 100 *Think of '15' and make it 100 times the size.*

100 times the size, 100 times the size

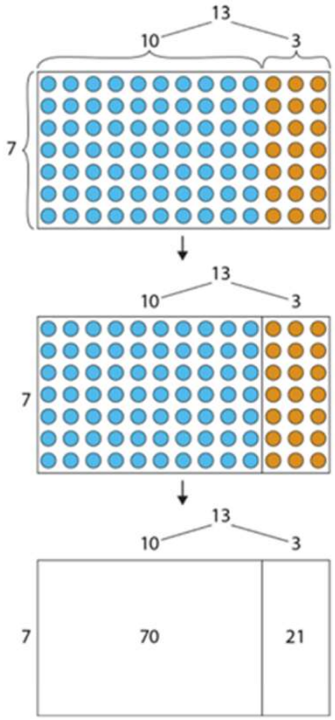
Ratio chart:

÷ 10	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
÷ 10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
	0	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500

÷ 100

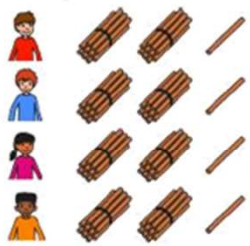
Maths | Modelling - Year 4: Multiplication and Division

Partitioning for multiplication

	Concrete and Pictorial Representations	Abstract
By applying knowledge of the distributive law (which means that if you split a number and multiply the split parts separately and add the separate answers together, you get the same answer as would get if you had multiplied the original number. The example here demonstrates this with 13 x 7. The 13 can be partitioned into a 10 and 3 with each of the partitions multiplied by 7. The products for those calculations can be added to find the final answer. **This strategy can be used both in written form and mentally, depending on the numbers involved and the strength of number fact knowledge.**		$13 \times 7 = 10 \times 7 + 3 \times 7$ $= 70 + 21$ $= 91$ $7 \times 13 = 7 \times 10 + 7 \times 3$ $= 70 + 21$ $= 91$

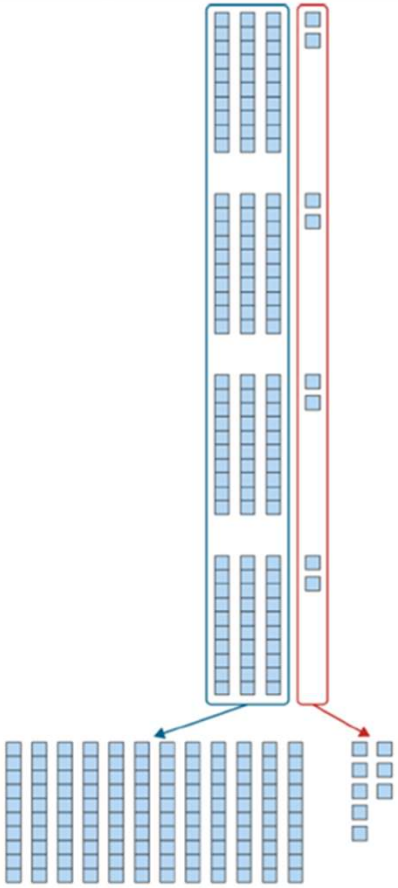
Maths | Modelling - Year 4: Multiplication and Division

Partitioning for division

	Concrete and Pictorial Representations	Abstract
Similar to multiplication, partitioning can be used to divide 2-digit numbers by single digit numbers. The example here shows how this can be done by splitting 84 into 8 tens and 4 ones which can each be divided by 4 and combined to reach the final answer. This is developed further later in KS2 when non-standard partitioning (splitting a number in a way other than by place value) can be used for efficient mental calculation. An example of this would be to solve 56 ÷ 4 by partitioning 56 into known multiples of 4 such as 40 and 16. Knowing that: 40 ÷ 4 = 10 and 16 ÷ 4 = 4 can allow the answer of 14 to efficiently calculated without the need for a written method.	*'Eighty-four sticks are shared equally between four children. How many sticks does each child get?'* 84 ÷ 4 = ?  Example solution reached by partitioning into tens and ones and dividing these separately. The quotients (answer for a division calculation) are then added.  8 tens ÷ 4 = 2 tens 4 ones ÷ 4 = 1 one 84 ÷ 4 = 21	

Maths | Modelling - Year 4: Multiplication and Division

Short multiplication

	Concrete and Pictorial Representations	Abstract
Short multiplication is a written method for multiplication taught as way to multiply a 2-digit number by a single digit number (such as 17 x 6). This is later extended to multiply 3 and 4-digit numbers by a single digit in Years 5 and 6. This method is initially introduced using physical resources such as dienes to represent what happens within the multiplication before moving to the written layout. It builds on their understanding of partitioning for multiplication explained above.		$32 \times 4 = 30 \times 4 + 2 \times 4$ $= 120 + 8$ • *'Three-tens-and-two-ones multiplied by four is equal to three tens multiplied by four and two ones multiplied by four.'* 3 tens × 4 = 12 tens 2 ones × 4 = 8 ones

Maths | Modelling - Year 4: Multiplication and Division

Example 1 – compact layout *with* place-value headings:

	10s	1s
	3	2
×		3
	9	6

- 3 × 2 ones = 6 ones
  'Write "6" in the ones column.'
- 3 × 3 tens = 9 tens
  'Write "9" in the tens column.'

Example 2 – compact layout *without* place-value headings:

	2	1
×		4
	8	4

- 4 × 1 one = 4 ones
  'Write "4" in the ones column.'
- 4 × 2 tens = 8 tens
  'Write "8" in the tens column.'

Maths | Modelling - Year 4: Multiplication and Division

When the product of one column is greater than 9, regrouping needs to be done. In this example, 3 x 4 gives the product of 12 ones which cannot fit in the one's column. Therefore, 10 ones are exchanged for 1 ten which is written below the tens column and the 2 is written in the ones column.

Example 1 – compact layout *with* place-value headings:

Step 1 – write the factors:

	10s	1s
	2	4
×		3

Step 2 – multiply the single-digit number by the ones and regroup:

	10s	1s
	2	4
×		3
		2
	1	

3 × 4 ones = 12 ones
= 1 ten + 2 ones

'Write "1" below the tens column and "2" in the ones column.'

Step 3 – multiply the single-digit number by the tens and add the tens from regrouping:

	10s	1s
	2	4
×		3
	7	2
	1	

3 × 2 tens = 6 tens

6 tens + 1 ten = 7 tens
'Write "7" in the tens column.'

Example 2 – compact layout *without* place-value headings:

	1	8
×		5
	9	0
	4	

- 5 × 8 ones = 40 ones; 40 ones = 4 tens and 0 ones
  'Write "4" below the tens column and '0' in the ones column.'
- 5 × 1 ten = 5 tens
  5 tens + 4 tens = 9 tens
  'Write "9" in the tens column.'

Maths | Modelling - Year 4: Multiplication and Division

Short division

	Concrete and Pictorial Representations	Abstract

Short division is a written method for division taught as way to divide a 2-digit number by a single digit number (such as 84 ÷ 4). This is later extended to divide by two-digit numbers in Years 5 and 6 but remains the most efficient written method for dividing by a single digit.

This method is initially introduced using physical resources such as dienes and pictorial representations to explore the method before moving to the written layout. It builds on their understanding of partitioning for division explained above.

Step 1 – write the divisor and dividend

10s 1s

$4\overline{)84}$

'Eighty-four divided by four.'

Step 2 – sharing the tens

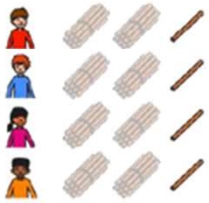
10s 1s

$4\overline{)84}$ (2)

8 tens ÷ 4 = 2 tens

'Eight tens divided by four is equal to two tens.'

Step 3 – sharing the ones



10s 1s

$4\overline{)84}$ (21)

8 tens ÷ 4 = 2 tens

4 ones ÷ 4 = 1 one

'Four ones divided by four is equal to one one.'

Summary

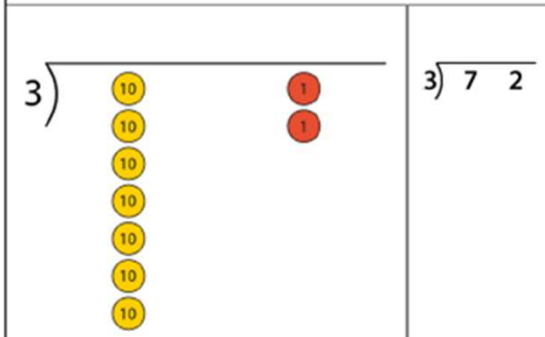
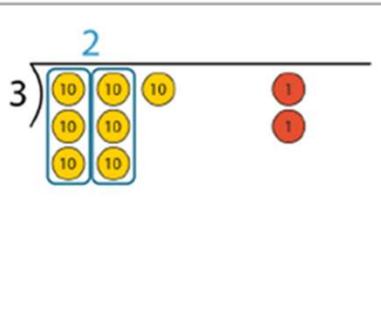
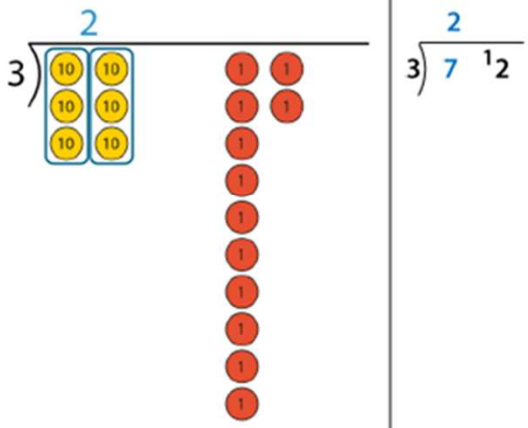
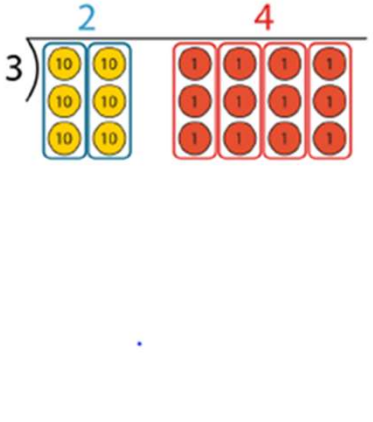
10s 1s

$4\overline{)84}$ (21)

'Each child gets twenty-one sticks.'

Maths | Modelling - Year 4: Multiplication and Division

When the division within a place value column leaves a remainder, exchanging is used.
In this example, when the 7 tens are divided by 3 there is a remainder of 1 ten. This is then exchanged for 10 ones giving a total of 12 ones to be divided next.

Step 1 – write the divisor and dividend	Step 2 – sharing the tens…
	 
'Seventy-two divided by three.'	7 tens ÷ 3 = 2 tens r 1 ten *'Write "2" in the tens column…'*
Step 3 – …and exchanging	**Step 4 – sharing the ones**
	 
1 ten = 10 ones *'…and write "1" to the left of the ones digit of the dividend to make twelve ones.'*	12 ones ÷ 3 = 4 ones *'Write "4" in the ones column.'*

Maths | Modelling - Year 5: Addition and Subtraction

Year 5 – addition and subtraction

Application of Number Facts

	Concrete and Pictorial Representations	Abstract
Number facts to 1,000,000 (These are explored both as additive and multiplicative equations and applied within the range of strategies listed below)		

10,000									
1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

100,000									
10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000

10,000	
5,000	5,000

10,000 = 5,000 × 2
10,000 ÷ 2 = 5,000

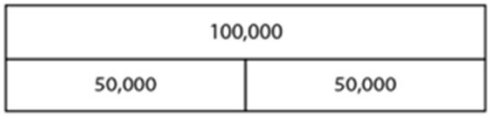
10,000			
2,500	2,500	2,500	2,500

10,000 = 2,500 × 4
10,000 ÷ 4 = 2,500

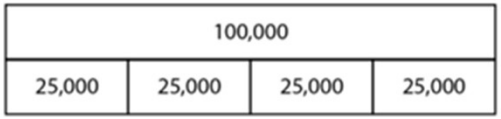
10,000				
2,000	2,000	2,000	2,000	2,000

10,000 = 2,000 × 5
10,000 ÷ 5 = 2,000

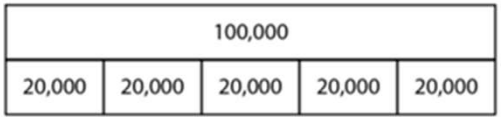
Maths | Modelling - Year 5: Addition and Subtraction



100,000 = 50,000 × 2
100,000 ÷ 2 = 50,000



100,000 = 25,000 × 4
100,000 ÷ 4 = 25,000



100,000 = 20,000 × 5
100,000 ÷ 5 = 20,000

Mental Strategies with 5 and 6-digit numbers

Each of the mental strategies taught in Year 3 and used in Year 4 with 4 -digit numbers are applied and explored within numbers in the tens of thousands and hundreds of thousands. This gives a chance for the children to be reminded of those strategies and gain familiarity in using them with increasing confidence while working with 5 and 6-digit numbers.

See the Year 3 section above for an explanation and example of each mental strategy covered.

Column Addition and subtraction

The column addition and subtraction algorithms taught in Y3 and extended to include 4-digit numbers in Y4 are further extended into the tens of thousands and hundreds of thousands. Examples of these can be seen below.

For an explanation of these methods and how they are introduced, see the Y3 section above.

Maths | Modelling - Year 5: Addition and Subtraction

Column addition and subtraction:

- With place-value headings

	Thousands			Ones		
	100s	10s	1s	100s	10s	1s
	3	6	5	0	0	0
+	2	1	4	0	0	0
	5	7	9	0	0	0

- Without place-value headings

```
  3 6 5,0 0 0
+ 2 1 4,0 0 0
  ---------
  5 7 9,0 0 0
  ---------
```

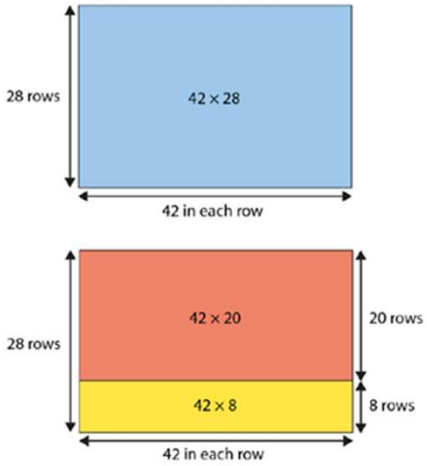
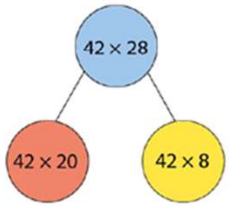
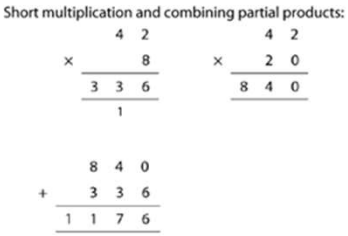
Maths | Modelling - Year 5: Multiplication and Division

Year 5 – multiplication and division

By the time they enter Year 5, children are expected to be able to confidently recall their times tables up to 12x12. This knowledge forms the foundation for learning more advanced methods and working with both larger numbers and with decimals.

Children in Year 5 are also taught a formal written method for multiplying and dividing by two digit numbers: long multiplication and long division.

Long Multiplication

	Concrete and Pictorial Representations	Abstract
Long multiplication is introduced by first using short multiplication by both the ones and tens separately. These answers (known as partial products) can then be added to find the product of the complete multiplication. This introduces the children to the fact that multiplication by a 2 digit number can be done by first multiplying by the ones, then the tens and these products then added.	Area model/grid:  Part–part–whole model: 	Short multiplication and combining partial products: 

Maths | Modelling - Year 5: Multiplication and Division

The separate short multiplication and addition steps are then combined within the long multiplication algorithm. To the right is shown the expanded layout. This most clearly shows each of the steps within the method. Children will quickly move from this expanded layout shown below once they are secure with the method.

Multiplication algorithm – expanded layout:

Step 1 – write the factors

	100s	10s	1s
		3	1
×		2	4

Step 2 – multiply the ones digit by the ones digit

	100s	10s	1s	
		3	1	
×		2	4	
			4	4 × 1 one = 4 ones

Step 3 – multiply the tens digit by the ones digit and regroup

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	4 × 3 tens = 12 tens = 1 hundred + 2 tens

Step 4 – place a zero to show that it's ten times the size

	100s	10s	1s
		3	1
×		2	4
	1	2	4
			0

Step 5 – multiply the ones digit by the tens digit

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	
		2	0	2 tens × 1 one = 2 tens

Step 6 – multiply the tens digit by the tens digit

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	
	6	2	0	2 tens × 3 tens = 6 hundreds

Step 7 – add the partial products

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	31 × 4
	6	2	0	31 × 20
	7	4	4	

Maths | Modelling - Year 5: Multiplication and Division

Multiplication algorithm – expanded layout:

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	31 × 4
	6	2	0	31 × 20
	7	4	4	

Multiplication algorithm – compact layout:

		3	1
×		2	4
	1	2	4
	6	2	0
	7	4	4

Maths | Modelling - Year 5: Multiplication and Division

Step 4 – subtract to find the remainder

$$\begin{array}{r} 0\ 1 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1 \\ \hline 1\ 2 \end{array}$$

(1 ten × 31 = 31 tens)

43 tens – 31 tens = 12 tens

- *'Write "12" underneath the "31".'*

Step 5 – exchange tens for ones and combine with the existing ones

$$\begin{array}{r} 0\ 1 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1\ \downarrow \\ \hline 1\ 2\ 4 \end{array}$$

(1 ten × 31 = 31 tens)

12 tens = 120 ones

120 ones + 4 ones = 124 ones

- *'Write "4" after the "12".'*

Step 6 – divide the ones

$$\begin{array}{r} 0\ 1\ 4 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1 \\ \hline 1\ 2\ 4 \\ 1\ 2\ 4 \end{array}$$

(1 ten × 31 = 31 tens)

(4 ones × 31 = 124 ones)

124 ones ÷ 31 = 4 ones

(refer to the ratio chart)

- *'Write "4" in the ones column of the answer line and write "124" underneath the "124", aligning the digits.'*

Step 7 – subtract to show there is no remainder

$$\begin{array}{r} 0\ 1\ 4 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1 \\ \hline 1\ 2\ 4 \\ 1\ 2\ 4 \\ \hline 0 \end{array}$$

(1 ten × 31 = 31 tens)

(4 ones × 31 = 124 ones)

124 ones – 124 ones = 0 ones

- *'Write "0" underneath the "31".'*

Maths | Modelling - Year 6: Addition and Subtraction

Year 6 – addition and subtraction

Application of Number Facts

	Concrete and Pictorial Representations	Abstract
Number facts to 100,000,000 (These are explored both as additive and multiplicative equations and applied within the range of strategies listed below)		

1,000,000	
500,000	500,000

$1{,}000{,}000 \div 2 = 500{,}000$

$1{,}000{,}000 \div 500{,}000 = 2$

$\frac{1}{2} \times 1{,}000{,}000 = 500{,}000$

$1{,}000{,}000 \times \frac{1}{2} = 500{,}000$

$2 \times 500{,}000 = 1{,}000{,}000$

$500{,}000 \times 2 = 1{,}000{,}000$

1,000,000			
250,000	250,000	250,000	250,000

$1{,}000{,}000 \div 4 = 250{,}000$

$1{,}000{,}000 \div 250{,}000 = 4$

$\frac{1}{4} \times 1{,}000{,}000 = 250{,}000$

$1{,}000{,}000 \times \frac{1}{4} = 250{,}000$

$4 \times 250{,}000 = 1{,}000{,}000$

$250{,}000 \times 4 = 1{,}000{,}000$

1,000,000				
200,000	200,000	200,000	200,000	200,000

$1{,}000{,}000 \div 5 = 200{,}000$

$1{,}000{,}000 \div 200{,}000 = 5$

$\frac{1}{5} \times 1{,}000{,}000 = 200{,}000$

$1{,}000{,}000 \times \frac{1}{5} = 200{,}000$

Maths | Modelling - Year 6: Addition and Subtraction

Mental Strategies with numbers in the millions

Each of the mental strategies taught in Year 3 and used in Year 4 and 5 with increasingly larger numbers are applied and explored within numbers in the millions. This gives a chance for the children to be reminded of those strategies and to explore how patterns within the small place values can also been seen in larger numbers (for example, that 4 x 250 = 1,000 and 4 x 250,000 = 1,000,000).

See the Year 3 section above for an explanation and example of each mental strategy covered.

Column Addition and subtraction

The column addition and subtraction algorithms taught in Y3 and used in Year 4 and 5 with increasingly larger numbers are further extended into the millions.

For an explanation of these methods and how they are introduced, see the Y3 section above.

Column addition:

$$\begin{array}{r} 643,801 \\ +\ 505,370 \\ \hline 1,149,171 \\ {\scriptstyle 1\ \ \ \ \ \ } \end{array}$$

Maths | Modelling - Year 6: Multiplication and Division

Long Division

	Concrete and Pictorial Representations	Abstract
The method for long division is taught to provide a formal written method when dividing by two-digit numbers that cannot be calculated mentally. Before attempting the written method, a ratio chart (frequently referred to as a 'What I Know' box of W.I.K.) must be created. This is a starter list of times tables for the divisor. It is not a complete list; it contains those that can be calculated quickly and acts as a starting point to work out the others if needed. The 2x, 4x, and 8x can be found by doubling, doubling and doubling again. The 10x can be found by using knowledge of place value and the 5x can be found by halving the 10x. The initial layout for long division is very similar to short division with the key difference being that the subtraction step is recorded, rather that being held mentally. This is to avoid errors which would likely occur if all of the steps of the division were held mentally.	Ratio chart: ×31 1 — 31 2 — 62 3 — 4 — 124 5 — 155 6 — 7 — 8 — 248 9 — 10 — 310	Step 1 – write the divisor, frame and dividend 31)4 3 4
	Step 2 – divide the hundreds 0 31)4 3 4 4 hundreds ÷ 31 = 0 hundreds r 4 hundreds • *'Write "0" in the hundreds column of the answer line.'*	Step 3 – exchange hundreds for tens, combine with the existing tens and divide… 0 1 31)4 3 4 3 1 (1 ten × 31 = 31 tens) 4 hundreds = 40 tens 40 tens + 3 tens = 43 tens 43 tens ÷ 31 = 1 ten and a remainder • *'Write "1" in the tens column of the answer line and write "31" underneath the "43".'*