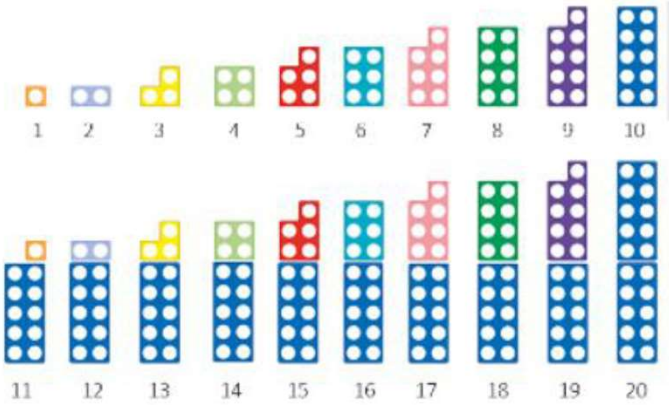
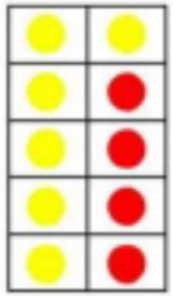
Maths | Modelling - Foundation Stage: Algebra

Reception Number and Number Patterns	
Objective/Strategy	**Concrete and Pictorial Modelling**
Have a deep understanding of number to 10, including the composition of each number.	  
Subitise (recognise quantities without counting) up to 5	 
Numerical Patterns	
Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.	      Can you think of any other ways to represent this number?  

Maths | Modelling - Year 1: Addition

Key Vocabulary

- Sum
- total
- parts and wholes
- Plus
- Add
- altogether
- more
- 'is equal to'
  'is the same as'

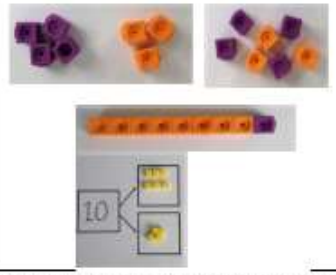
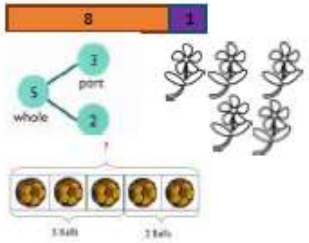
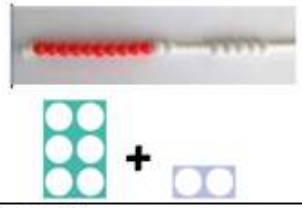
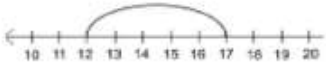


6 + 4 = 10
4 + 6 = 10
10 – 4 = 6
10 – 6 = 4

Year 1 Addition

within 10 and then 20

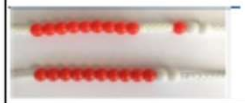
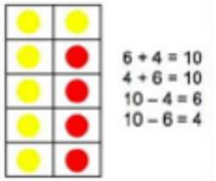
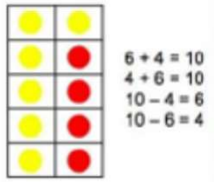
Use dienes or base ten (tens rods and ones/units cubes) for number in preparation for Year 2

Objective/Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model Addition Year 1	Use part-whole model. Use cubes to add two numbers together as a group or in a bar. 	Use pictures to add two numbers together as a group or in a bar. 	Use the part-whole diagram as shown below to move into the abstract. Include missing number questions to support varied fluency. 8 = 5 + 3 5 + 3 = 8  8 = ? + 3 5 + ? = 8
Starting at the bigger number and counting on Addition Year 1	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. Use counters, tens frames or Numicon. 	Start at the larger number on the number line and count on in ones or in one jump to find the answer.  **12** + 5= 17	Place the larger number in your head and count on the smaller number to find your answer. 5 + **12** = 17
Regrouping to make 10 This is an essential skill for column addition later Addition Year 1	6 + 5 = 11 Start with the bigger number and use the smaller number to make 10. Use ten frames. 	Use pictures or a number line. Regroup or partition the smaller number using the part-whole model to make 10. 	7 + 4= 11 3+1 I am at seven, how many more do I need to make 10? (3) How many more do I add on now? (1)

Maths | Modelling - Year 1: Addition continued

Key Vocabulary

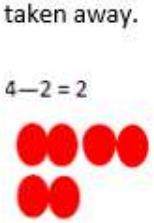
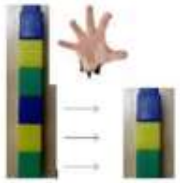
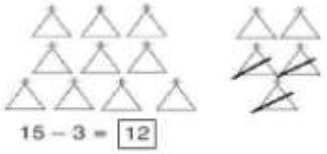
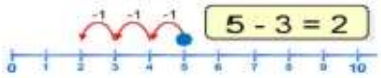
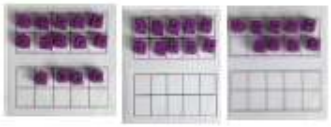
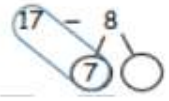
- Sum
- total
- parts and wholes
- Plus
- Add
- altogether
- more
- 'is equal to'
  'is the same as'

	9+3=12 This example shows how bead strings can be used to demonstrate the same method. 		
Use number bonds to 10 to make number bonds to 20 Addition Year 1	Use 10s frames and coloured counters (1 frame for the number bond to 10 and 2 frames for the number bond to 20) and Numicon. 	Colour in dots (2 different colours to make the bonds) on 10s frames (1 frame for the number bond to 10 and 2 frames for the number bond to 20). 	7+3=10 so 17+3=20 or 13+7=20

Maths | Modelling - Year 1: Subtraction

Key Vocabulary

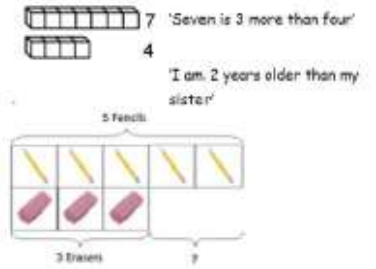
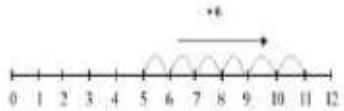
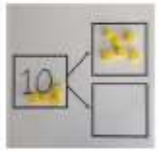
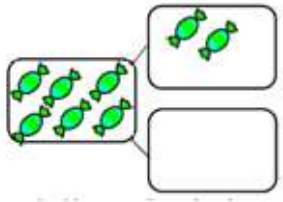
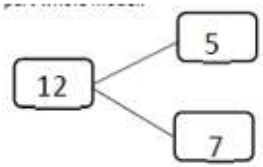
- take away
- less than
- the difference
- subtract
- minus
- fewer
- decrease

Year 1 Subtraction Use dienes (tens and ones/units) for number in preparation for year 2			
Objective/Strategy	**Concrete**	**Pictorial**	**Abstract**
Taking away 1-digit then 2-digit numbers up to 20 (start by not crossing 10, then crossing 10) Subtraction Year 1	Use physical objects, counters, cubes, bead strings, Numicon, etc. to show how objects can be taken away. 4—2 = 2   6 -3=3  20 – 7 = ___	Cross out drawn objects to show what has been taken away, count back on a number line, bar model.  15 – 3 = 12  5 - 3 = 2	7—4 = 3 16—9 = 7 Put 13 in your head, count back 4. What number are you at?
Make 10 when counting back to cross over 10 Subtraction Year 1	14—5= Make 14 on the ten frame with counters. Take 4 away to make ten, then take one more away so that you have taken 5. 	Use a number line, jump back 3 first, then another 4. Use ten as the stopping point. Use part-whole model.  	16—8 How many do we take off first to get to 10? How many left to take off?
Find the difference (relate to addition, counting on as well as counting back)	Compare objects and amounts Lay objects to represent bar model.	Count on using a number line to find the difference.	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?

Maths | Modelling - Year 1: Subtraction

Key Vocabulary

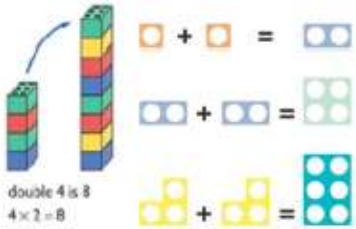
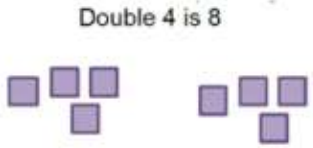
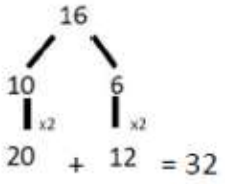
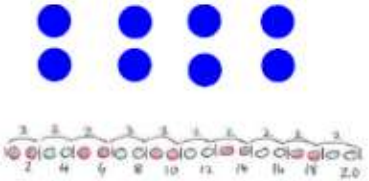
- take away
- less than
- the difference
- subtract
- minus
- fewer
- decrease

Subtraction Year 1			
Represent and use number bonds and related subtraction facts within 20. Include subtracting zero. Part-whole model Subtraction Year 1	Link to addition. 12+1=13 13-1=12 Use 10s frames and 2 different coloured counters to model inverse.  If 10 is the whole and 6 is one of the parts, what is the other part? 10 – 6 = 4	Use part-whole model (dienes drawn), bar model and draw dots in 10s frames. 	Move to using numbers within the part-whole model. Include missing number problems: 12 - ? = 5 7 = 12 - ? 

Maths | Modelling - Year 1: Multiplication

Key Vocabulary

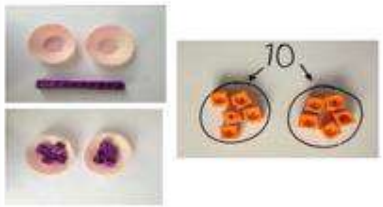
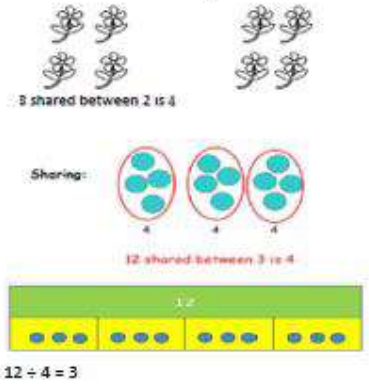
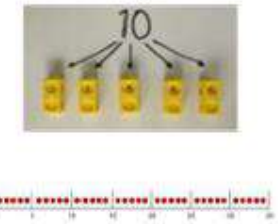
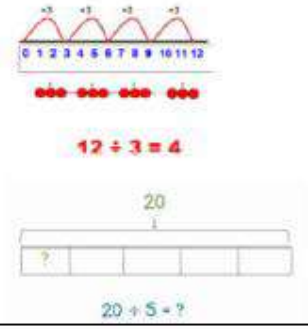
- Double
- Times
- multiplied by
- the product of
- groups of
- lots of
- equal groups

Year 1 Multiplication			
Objective/Strategy	**Concrete**	**Pictorial**	**Abstract**
Doubling Multiplication Year 1	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling. 	Draw pictures to show how to double numbers. 	Partition a number and then double each part before recombining it back together. 
Counting in multiples (2s, 5s, 10s) Multiplication Year 1	Count the groups of 2, 5 or 10 using bead strings, number lines, 100 square, Numicon, looking at images of groups. Children could use their fingers as they are counting. 	Children draw representations to show counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Maths | Modelling - Year 1: Division

Key Vocabulary

- Double
- Times
- multiplied by
- the product of
- groups of
- lots of
- equal groups

Year 1 Division			
Objective/Strategy	**Concrete**	**Pictorial**	**Abstract**
Division as sharing (children do not need to be familiar with the symbol yet) Division Year 1	I have 10 cubes, can you share them equally in 2 groups? 	Children use pictures and bar models to share quantities. E.g draw pictures of sharing 10 muffins between 2 plates. 	12 shared between 3 is 4. (12 ÷ 3 = 4)
Introduce division as grouping (children do not need to be familiar with the symbol yet) Division Year 1	Divide quantities into equal groups e.g. 20 counters in total how many equal groups of 2 can you make? Use cubes, counters, objects or place value counters to aid understanding. 	If you have a total of 12, how many equal groups of 3 will you have? Use a number line or a bar model. 	Divide 28 into 4s. How many equal groups do you get? (28 ÷ 4 =)

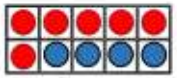
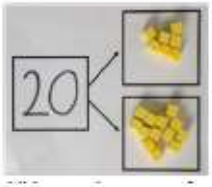
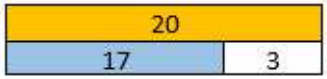
Maths | Modelling - Year 2: Addition

Key Vocabulary

- Sum
- Total
- parts and wholes
- Plus
- Add
- Altogether
- More
- 'is equal to'
- 'is the same as'

Year 2 Addition

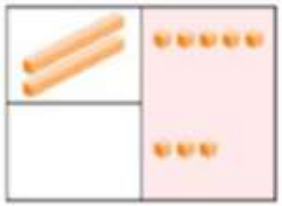
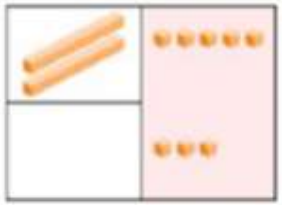
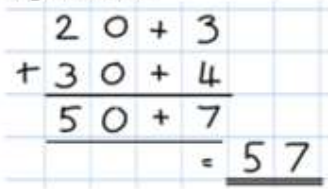
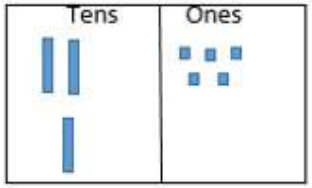
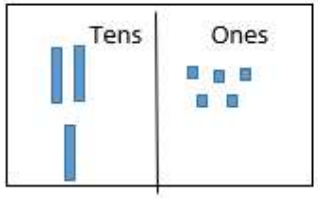
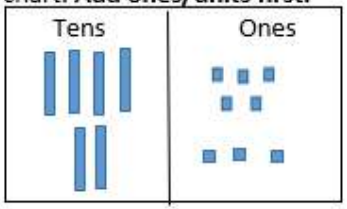
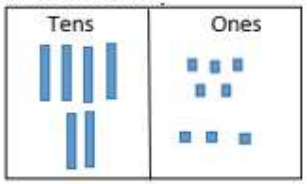
up to 100 and starting to cross over 100

Objective/Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten to make 100 and numbers up to 100. Addition Year 2	Model creating number bonds using a 10s frame to represent 100 and 2 different coloured counters, dienes or Numicon 10s.  (Each counter represents 10 in this example) 60 + 40 = 100 	Drawing of 10s rods, 10s numberline or 10s place value counters on a 10s frame to represent 100.  60 + 40 = 100	20 + 30 = 50 70 = 50 + 20 40 + □ = 60 Recite number bonds to 100
Use known addition number facts to make 20 to explore **subtraction** facts (to make 20 and numbers up to 20) links to Year 2 subtraction Addition Year 2	Children explore ways of making numbers within 20 using part-whole model, counters or cubes and 2 tens frames with 2 different colour counters. 	Use bar model, number lines, part-whole model with numbers, drawings of 10s frames with dots.   	Explore commutativity of addition and make list. E.g. 12+3=15 15=12+3 3+12=15 15=3+12 Explore the concept of the inverse relationship of addition and subtractions and use this to check calculations. E.g. 12+3=15 So 15-3=12 and 15-12=3

Maths | Modelling - Year 2: Addition

Key Vocabulary

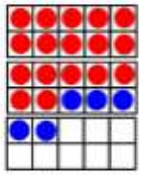
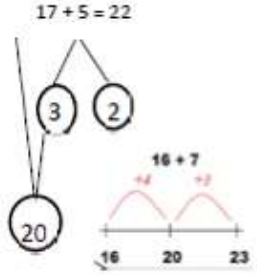
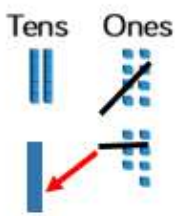
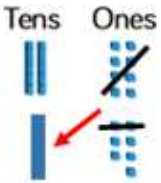
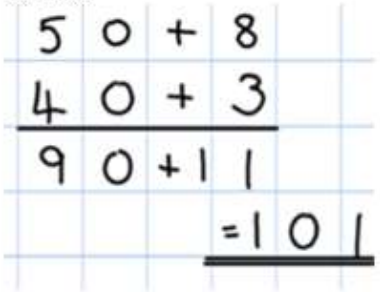
- Sum
- Total
- parts and wholes
- Plus
- Add
- Altogether
- More
- 'is equal to'
- 'is the same as'

Add a two-digit number and ones (not bridging 10s, so no exchanging) Addition Year 2 See also Empty Number line method as an alternative (below)	Use dienes and place value chart. **Add ones/units first.** 	Use part-whole model or number track to model. When bridging 10s use a number line, draw dienes in place value chart. Add ones/units first. 	Lead into recording in column format, to reinforce place value and prepare children for formal written methods with larger values. 
Add a 2-digit number and tens Addition Year 2	25 + 10 = 35 Explore that the ones digit does not change. Use dienes in a place value chart. **Add ones/units first.** 	Draw dienes in the place value chart to help to lead into column addition. Look at ones/units column first to see if there's anything to add.  25 + 10 = 35	27 + 10 = 37 27 + 20 = 47 27 + ☐ = 57
Add two, 2-digit numbers (no exchanging) Addition Year 2	45 + 23 = Use dienes in a place value chart. **Add ones/units first.** 	Draw dienes in the place value chart to help to lead into column addition. Add ones then add tens. 	27 + 31 = 58 31 + ☐ = 58

Maths | Modelling - Year 2: Addition

Key Vocabulary

- Sum
- Total
- parts and wholes
- Plus
- Add
- Altogether
- More
- 'is equal to'
- 'is the same as'

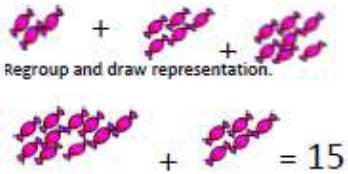
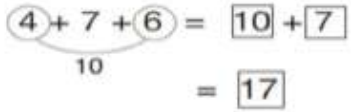
	Add the ones then add the tens.		
Regrouping to make 10 *This is an essential skill for column addition* Addition Year 2	Start with the bigger number and use the smaller number to make 10. Use ten frames.  17 + 5 = 22 17 + 3 = 20 20 + 2 = 22 Explore the pattern 17 + 5 = 22 27 + 5 = 32	Regroup or partition the smaller number using the part-whole model to make 10, a number line, 10s frames. 	Regroup to make other multiples of 20 + E.g. 33+9= 33+7+2=
Add two numbers **exchanging** 1s for 10s Addition Year 2	Model using dienes on a place value chart, exchange ten 1s for a tens rod and move that with the 10s. **Add ones/units first.** 28+7= 	Model drawing dienes on a place value chart, exchange ten 1s for a tens rod and move that with the 10s by crossing out the ten 1s and drawing the exchanged 10 under the tens column. 28+7= 	Use expanded column addition format. 

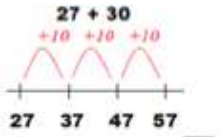
Maths | Modelling - Year 2: Addition

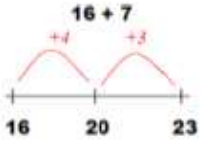
Key Vocabulary

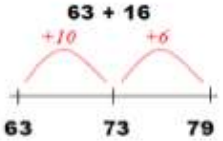
- Sum
- Total
- parts and wholes
- Plus
- Add
- Altogether
- More
- 'is equal to'
- 'is the same as'

<table>
<tr>
<td>Add three 1-digit Numbers

Addition Year 2</td>
<td>Combine to make 10 first if possible, or bridge 10 then add third digit. Use 3 different coloured counters, cubes.
  </td>
<td colspan="2"> 
</td>
<td>Combine the two numbers that make/ bridge ten then add on the third.
 
</td>
</tr>
<tr>
<td>Add with an empty number line

Addition Year 2</td>
<td colspan="2">Add multiples of 10
 

Add 1-digit numbers
 

Add pairs of 2-digit numbers
 
</td>
<td>For concrete support use a 100 square alongside or dienes.

For the pictorial stage, add on ready drawn number lines.</td>
<td>Independently use the empty number line method (apparatus can still be used alongside).</td>
</tr>
</table>

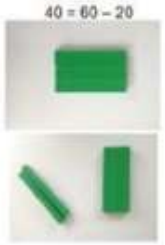
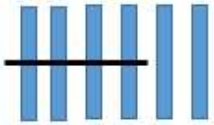
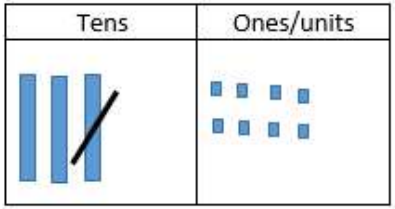
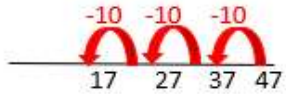
Maths | Modelling - Year 2: Subtraction

Key Vocabulary

- take away
- less than
- the difference
- subtract
- Minus
- Fewer
- Decrease

Year 2 Subtraction

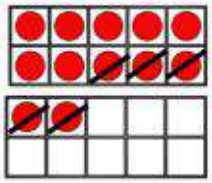
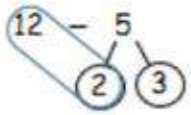
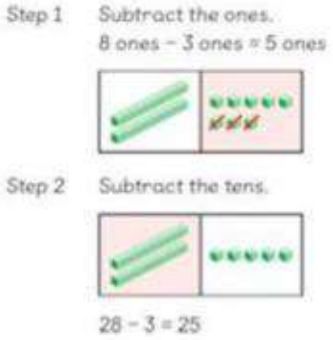
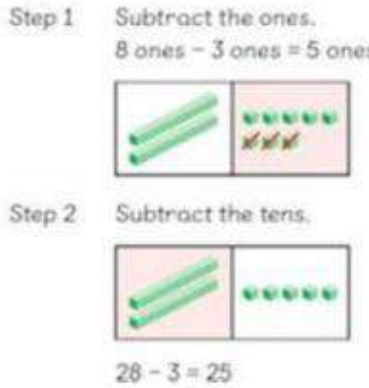
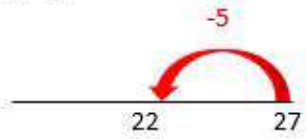
Subtraction number facts included in Year 2 Addition

Objective/Strategy	Concrete	Pictorial	Abstract
Subtracting multiples of ten to make 100 and numbers up to 100 Subtraction Year 2	Model using a 10s frame to represent 100 and 2 different coloured counters to create number bonds, dienes or Numicon 10s. 	Drawing of 10s rods and cross out 10s, 10s numberline or 10s place value counters on a 10s frame to represent 100. 	30 - 20= 10 70 = 100 - 30 90 - ☐ = 60 Recite subtraction number bonds to 100
Subtract multiples of 10 from numbers to 100 Subtraction Year 2 See also Empty Number line method as an alternative (below)	Use a place value chart with dienes (start to lead into column subtraction). **Subtract ones/units first.** 38 – 10 = 	Use a place value chart and draw dienes (start to lead into column subtraction). 38 – 10 = 	Subtract multiples of ten 47 – 30 = 
Make 10 when counting back to cross over 10 Subtraction Year 2	Use 10s frames to subtract back to the 10 by partitioning the second number. 12 -5=	Use number line and part-whole model to subtract back to the 10 by partitioning the second number.	Use the strategy mentally e.g. 23 – 5 = so 23 – 3 = 20, then there's 2 left to subtract (because 3 + 2 = 5) so 20 – 2 = 18

Maths | Modelling - Year 2: Subtraction

Key Vocabulary

- take away
- less than
- the difference
- subtract
- Minus
- Fewer
- Decrease

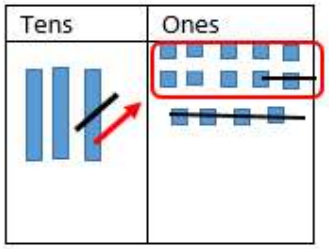
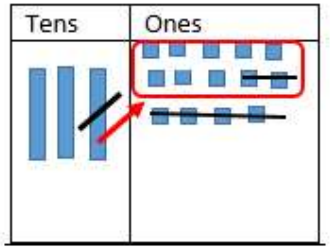
	 12 – 2 = 10 10 – 3 = 7	13 – 5 =  12 – 5 = 	
Subtract a 1-digit number from a 2-digit (no exchanging) Subtraction Year 2	Use dienes, subtract by moving ones away. **Subtract ones/units first.** 	Use drawings of dienes in a place value chart and cross out to subtract. 	27 – 5 = 

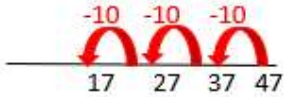
Maths | Modelling - Year 2: Subtraction

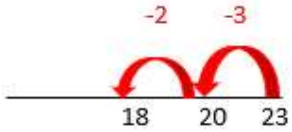
Key Vocabulary

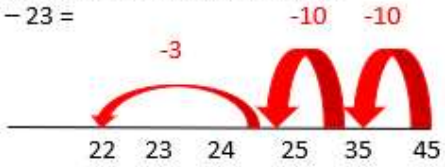
- take away
- less than
- the difference
- subtract
- Minus
- Fewer
- Decrease

<table>
<tr>
<td>Subtract two 2-digit numbers (exchanging 10) Included in Year 2

Subtraction Year 2</td>
<td>Use dienes, exchange a 10 for ten 1s/units. Subtract ones/units first.
34 – 16 =
 
</td>
<td>Use drawings of dienes in a place value chart and to exchange 10, cross out 10s rod and draw ten ones then cross out amount to subtract.
 
</td>
<td></td>
</tr>
<tr>
<td>Subtract with an empty number line

Subtraction Year 2</td>
<td>Subtract multiples of ten
47 – 30 =
 

Subtract 1-digit numbers
23 – 5 =
 

Subtract pairs of 2-digit numbers
45 – 23 =
 
</td>
<td>For concrete support use a 100 square alongside or dienes.

For the pictorial stage, subtract on ready drawn number lines.</td>
<td>Independently use the empty number line method (apparatus can still be used alongside).</td>
</tr>
</table>

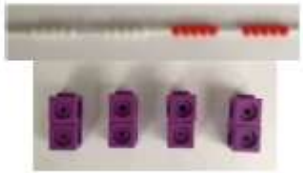
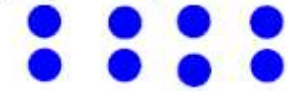
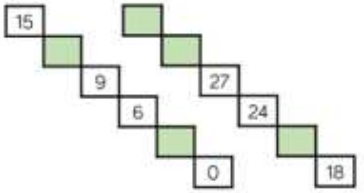
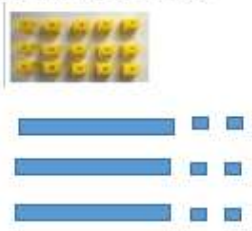
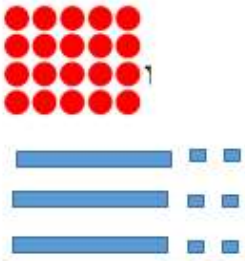
Maths | Modelling - Year 2: Multiplication

Key Vocabulary

- Double
- Times
- multiplied by
- the product of
- groups of
- lots of
- equal groups

Year 2 Multiplication

Children should be able to recall and use the multiplication and division facts for 2, 5 and 10 x table
Links between multiplication and division in the division section

Objective/Strategy	Concrete	Pictorial	Abstract
Counting in multiples of 3 (recap 2, 5 and 10) Multiplication Year 2	Count the groups of 2, 5, 10 and 3s using bead strings, number lines, 100 square, Numicon, looking at images of groups. 	Draw number lines counting in groups for hops.  	Count in multiples of a number aloud. Write sequences with multiples of numbers (fill in missing numbers from pattern). 0, 3, 6, 9, 12, 15 
Using arrays to solve multiplication calculations Multiplication Year 2	Use objects including dienes laid out in arrays to find the answers to 3 x 5, 3 x 12 etc. 	Draw representations of arrays to solve multiplication calculations. 	Write different calculations for an image of an array including + and x.  5 + 5 + 5 + 5 = 20 4 + 4 + 4 + 4 + 4 = 20 4 x 5 = 20 5 x 4 = 20

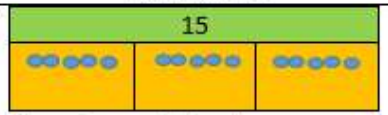
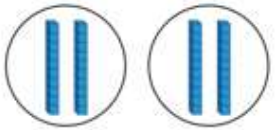
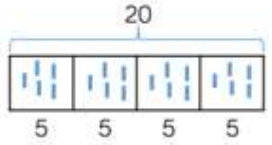
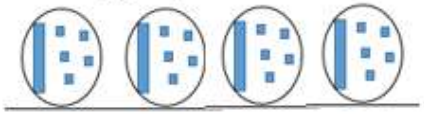
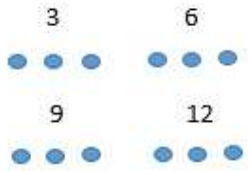
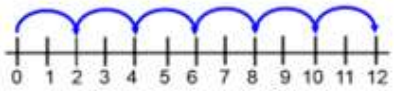
Maths | Modelling - Year 2: Division

Key Vocabulary

- Share
- group
- Divide
- divided by
- half

Year 2 Division

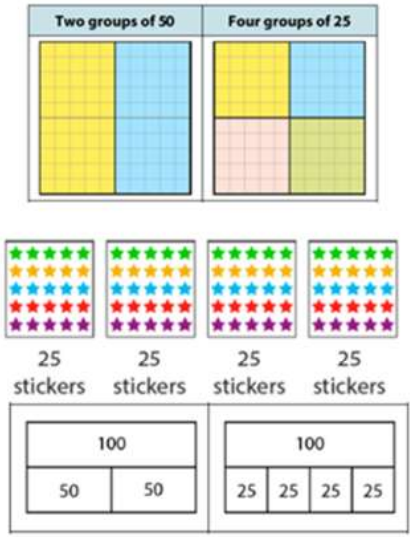
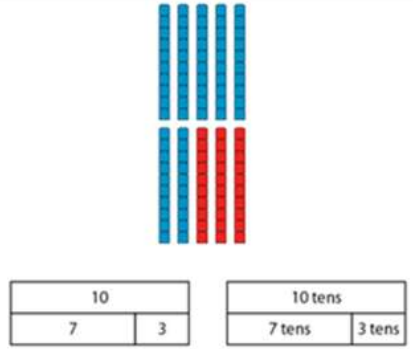
Children should be able to recall and use the multiplication and division facts for 2, 5 and 10 x table

Objective/Strategy	Concrete	Pictorial	Abstract
Division as sharing (with ÷ symbol) Division Year 2	 Use a bar model or draw groups to solve division calculations with counters, cubes or dienes.  40 ÷ 2 = 20	Draw a bar model/draw pictures to solve division calculations.  60 ÷ 4 =  Children will need to exchange 2 tens for 20 ones/units so they can put 1 ten and 5 ones in each group.	How many different ways can you divide/share equally 20? 20 ÷ 1 = 20 20 ÷ 20 = 1 20 ÷ 2 = 10 20 ÷ 10 = 2 20 ÷ 4 = 5 20 ÷ 5 = 4
Division as grouping (with ÷ symbol) Division Year 2	Divide quantities into equal groups e.g. 12÷3=; get 12 counters/cubes divide them into 3s. How many groups are there? 	Use bar modelling and a number line to aid solving division problems by grouping. 	How many groups of 4 in 24? 24 ÷ 4 = ? 24 ÷ ? = 4

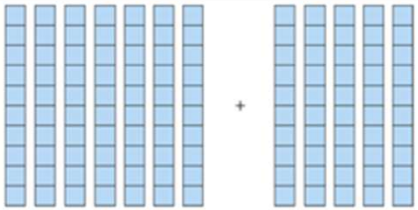
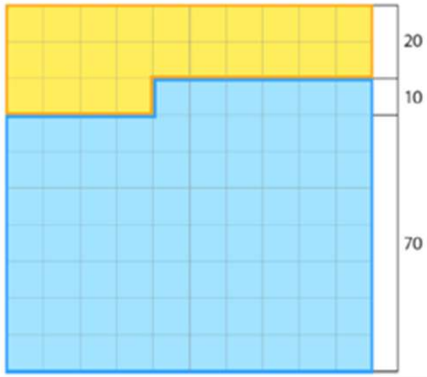
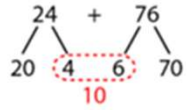
Maths | Modelling - Year 3: Addition and Subtraction

Year 3 – addition and subtraction

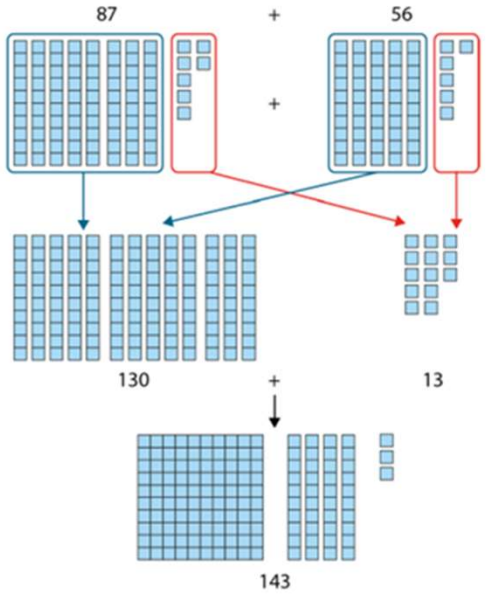
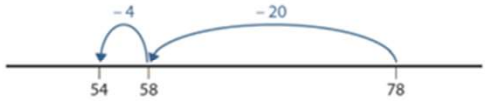
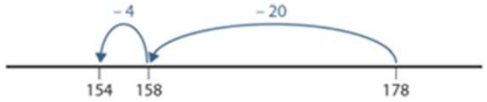
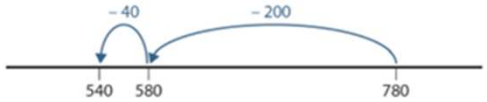
Application of Number Facts

	Concrete and Pictorial Representations	Abstract
Number facts to 100 (These are explored both as additive and multiplicative equations) **Stem Sentences**		100 = 50 + 50 100 = 2 × 50 100 = 50 × 2 100 ÷ 2 = 50 100 ÷ 50 = 2 100 = 25 + 25 + 25 + 25 100 = 4 × 25 100 = 25 × 4 100 ÷ 4 = 25 100 ÷ 25 = 4 100 = 25 + □ + 25 + 25 100 = 50 + □ 100 − 25 = □ 100 − 50 = □ 100 = 4 × □ 2 × □ = 100 100 ÷ 4 = □ □ = 100 ÷ 2 100 − 20 − 20 = □ □ = 100 − 10 − 10 − 10
Known addition number facts with single digit numbers can be used to calculate complements to 100 and add and subtract across 100		7 + 3 = 10 10 − 3 = 7 70 + 30 = 100 100 − 30 = 70

Maths | Modelling - Year 3: Addition and Subtraction

	 • 'I know that seven plus five is equal to twelve.' • 'So seven tens plus five tens is equal to twelve tens.' • 'Seventy plus fifty is equal to one hundred and twenty.'	7 + 5 = 12 7 tens + 5 tens = 12 tens 70 + 50 = 120
Partitioning **Addition of complements to 100 can be done by partitioning both addends (the numbers being added together) into tens and ones**		24 + 76 20 + 70 = 90 4 + 6 = 10 90 + 10 = 100 

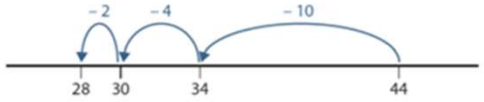
Maths | Modelling - Year 3: Addition and Subtraction

Partitioning can be used to add both two-digit and three-digit numbers	Dienes:  	Jottings: 
Partitioning the subtrahend (the number being taken away) can be used when subtracting. This method can be used either without bridging or with bridging.	Partitioning the subtrahend (without bridging): 78 – 24  178 – 24  780 – 240  	

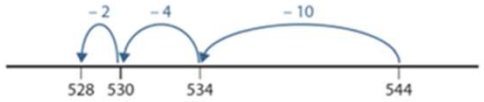
Maths | Modelling - Year 3: Addition and Subtraction

Partitioning the subtrahend (with bridging):

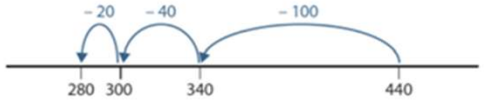
44 – 16



544 – 16



440 – 160

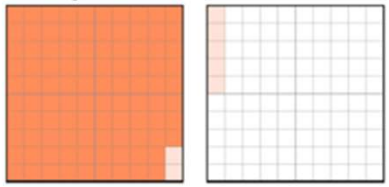


Making 100

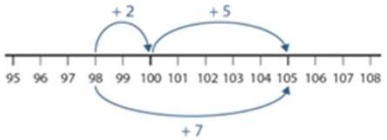
Bridging 100 (adding or subtracting across 100) can be done by first making 100

98 + 7

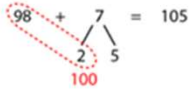
Hundred grids:



Number line:



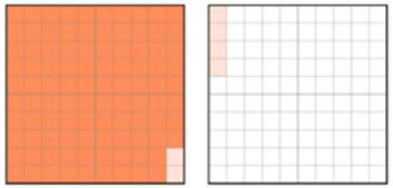
Jotting and equations:



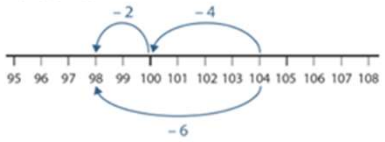
98 + 7 = 98 + 2 + 5
= 100 + 5
= 105

104 – 6

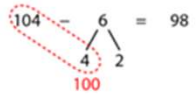
Hundred grids:



Number line:



Jotting and equations:



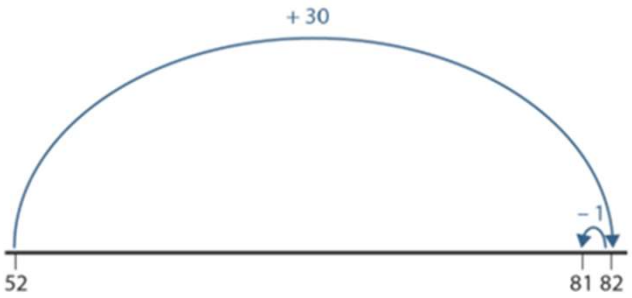
104 – 6 = 104 – 4 – 2
= 100 – 2
= 98

Maths | Modelling - Year 3: Addition and Subtraction

Adjusting

Adjusting is a more efficient addition strategy than partitioning when one of the numbers involved is close to a multiple of 10 or 100 (e.g. 49 is close to 50).

In the example given, 30 is added rather than 29 as it is a simpler calculation. 1 is then subtracted to adjust for the extra 1 that was added.



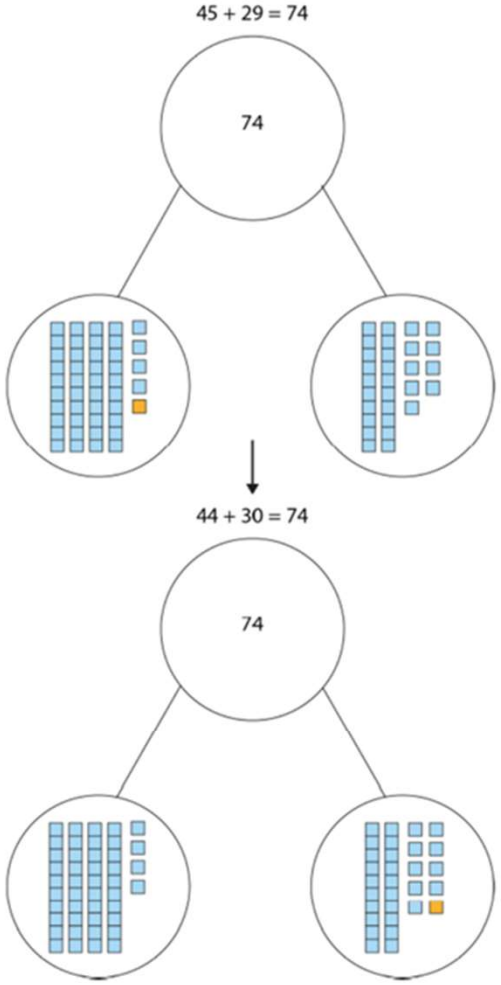
$$\begin{aligned} 52 + 29 &= 52 + 30 - 1 \\ &= 82 - 1 \\ &= 81 \end{aligned}$$

Maths | Modelling - Year 3: Addition and Subtraction

Redistributing

In the redistribution strategy an addition calculation is made simpler by increasing one addend and decreasing the other by the same amount.

There are similarities between the redistributing strategy and the adjusting strategy. However, with redistribution, the total remains the same at all times whereas with adjusting the total amount is increased to simplify the calculation and then decreased again.



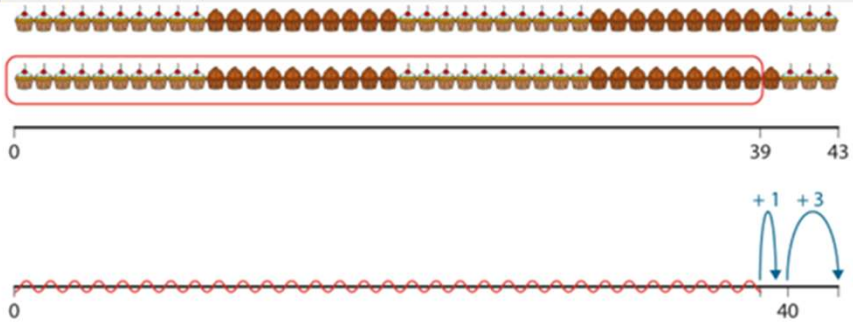
45 + 29 = 44 + 30 = 74

Maths | Modelling - Year 3: Addition and Subtraction

Finding the difference (adding on)

In this strategy, start with the subtrahend and add on to reach the minuend. The amount needed to be added will be the difference and the answer to the calculation.

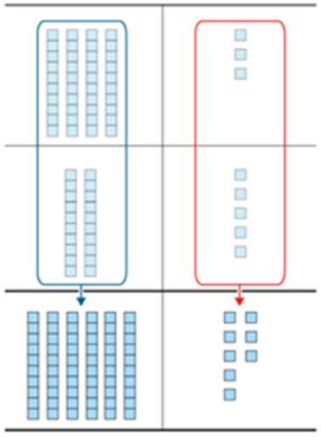
This strategy is particularly useful when the minuend and subtrahend are close together (e.g. 43 – 39)



$43 - 39 = 4$

Column Addition

Column addition is the formal written method for addition taught and used throughout KS2 and beyond for times when an efficient mental method is either not known or cannot be used to a suitable degree of accuracy.

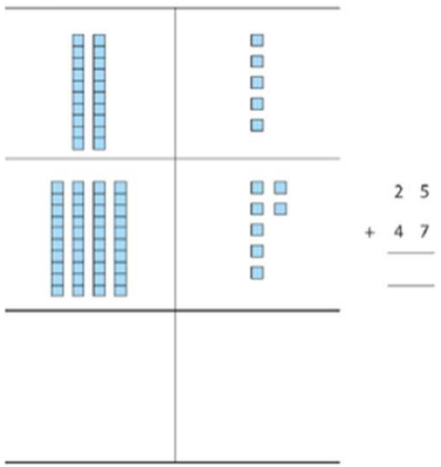


$$\begin{array}{r} 43 \\ +\ 25 \\ \hline 68 \end{array}$$

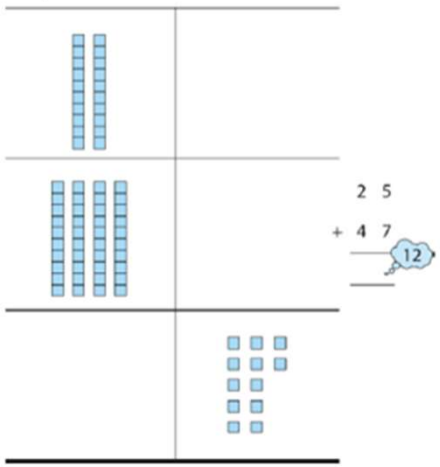
Maths | Modelling - Year 3: Addition and Subtraction

When the total of any column is 10 or greater, we must regroup. In the example shown, this involves exchanging 10 ones within the number 12 for 1 ten. This leaves 2 in the ones column and 1 ten below the tens column to be added when the tens are added.

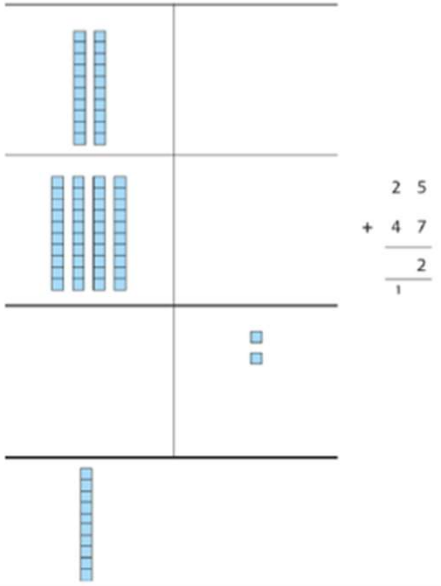
Step 1



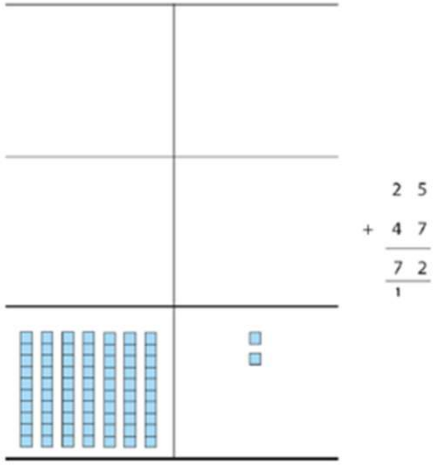
Step 2



Step 3



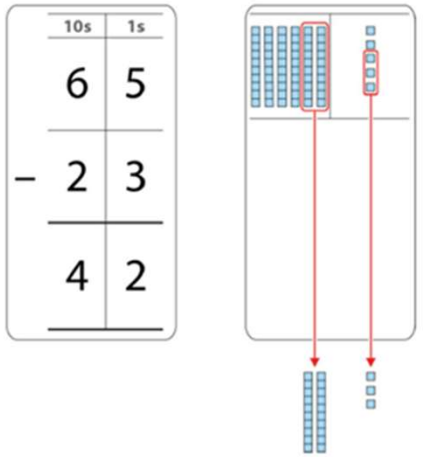
Step 4



Maths | Modelling - Year 3: Addition and Subtraction

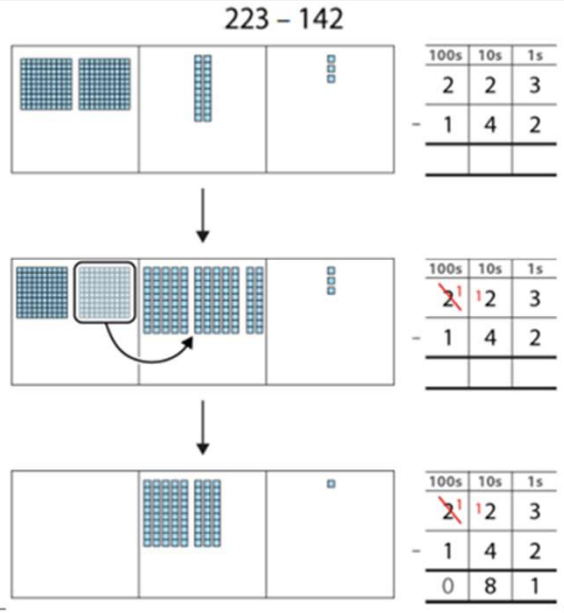
Column Subtraction

Similar to column addition, columns subtraction is the formal written method for subtraction taught and used throughout KS2 and beyond for times when an efficient mental method is either not known or cannot be used to a suitable degree of accuracy.



When the subtrahend (the number on the second row) in any column is greater than the minuend above it (the number at the top), we must regroup.

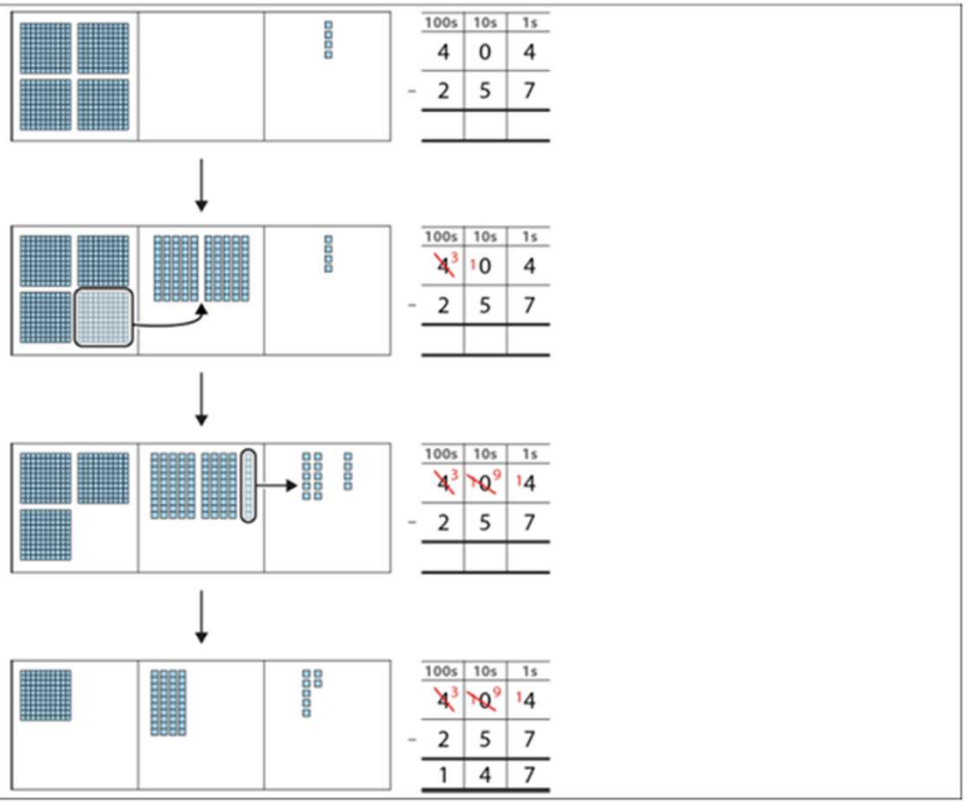
In the example shown (where 4 tens cannot be taken away from 2 tens), this involves exchanging 1 of the hundreds for 10 tens leaving 1 hundred remaining in the hundreds column and combining the exchanged 10 tens with the existing 2 tens to give 12 tens in the tens column. This allows 4 tens to be taken away from the 12 tens.



Maths | Modelling - Year 3: Addition and Subtraction

Regrouping can sometimes require working through a column with zero because the zero shows there is nothing to be exchanged.

In this situation, as shown in the example, regrouping can be done by exchanging from the next column to the left (the hundreds in this case). The regrouping must first be done into the column with zero (so exchanging 1 hundred into ten tens) which can then lead to regrouping into the column where the initial subtraction wasn't possible (so exchanging 1 of the previously exchanged tens into 10 ones).



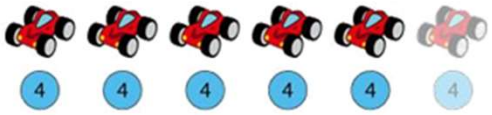
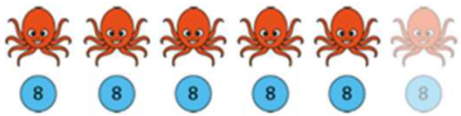
Maths | Modelling - Year 3: Multiplication and Division

Year 3 – multiplication and division

Within Year 3, the children continue to develop their times table knowledge by recalling the 5x and 2x tables learnt in KS1 and learning their times table number facts for the 4x and 8x tables. The 2x, 4x and 8x tables are taught in this sequence to reinforce the doubling relationship between them. Once the 2x, 4x and 8x tables are secure, the children then learn the 3x, 6x and 9x tables and the relationship between them.

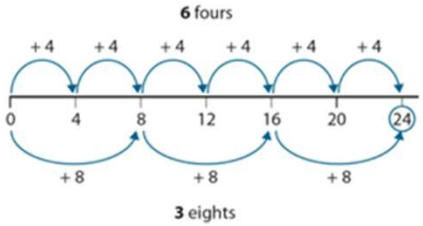
While a formal written method for multiplication and division is not taught in Year 3, the acquisition of times table knowledge is essential for the children to be ready to learn these in Year 4.

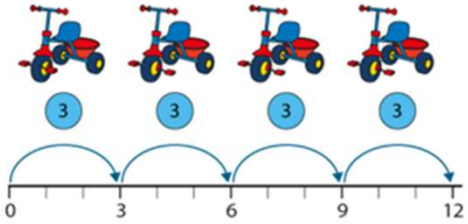
Number Facts

	Concrete and Pictorial Representations	Abstract
Times table number facts: 5x and 2x table (This is a recap of KS1 learning)		
Times table number facts: 2x, 4x and 8x table and the relationship between them **STEM SENTENCES**	 **Number of cars / Total number of wheels** 0 → 0 1 → 4 2 → 8 3 → 12 4 → 16 5 → 20 **6 → 24**  **Number of octopuses / Total number of tentacles** 0 → 0 1 → 8 2 → 16 3 → 24 4 → 32 5 → 40 **6 → 48**	$0 \times 4 = 0$ $1 \times 4 = 4$ $2 \times 4 = 8$ $3 \times 4 = 12$ $4 \times 4 = 16$ $5 \times 4 = 20$ $\mathbf{6 \times 4 = 24}$ $4 \times 0 = 0$ $4 \times 1 = 4$ $4 \times 2 = 8$ $4 \times 3 = 12$ $4 \times 4 = 16$ $4 \times 5 = 20$ $\mathbf{4 \times 6 = 24}$ $0 \times 8 = 0$ $1 \times 8 = 8$ $2 \times 8 = 16$ $3 \times 8 = 24$ $4 \times 8 = 32$ $5 \times 8 = 40$ $\mathbf{6 \times 8 = 48}$ $8 \times 0 = 0$ $8 \times 1 = 8$ $8 \times 2 = 16$ $8 \times 3 = 24$ $8 \times 4 = 32$ $8 \times 5 = 40$ $\mathbf{8 \times 6 = 48}$

Maths | Modelling - Year 3: Multiplication and Division

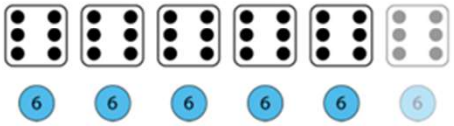
Times table number facts: 3x, 6x and 9x table and the relationship between them





Number of tricycles	Total number of
0	0
1	3
2	6
3	9
4	12
5	15
6	18

0 × 3 = 0	3 × 0 = 0
1 × 3 = 3	3 × 1 = 3

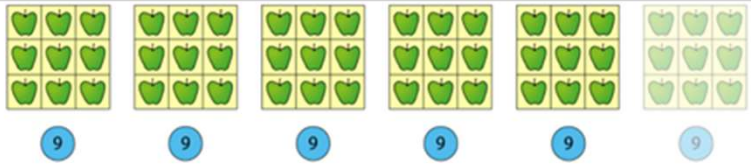


Number of six-value dice	Total number of dots
0	0
1	6
2	12
3	18
4	24
5	30
6	**36**

0 × 6 = 0	0 × 6 = 0
1 × 6 = 6	1 × 6 = 6
2 × 6 = 12	2 × 6 = 12
3 × 6 = 18	3 × 6 = 18
4 × 6 = 24	4 × 6 = 24
5 × 6 = 30	5 × 6 = 30
6 × 6 = 36	**6 × 6 = 36**

Number	**0**	1	2	**3**	4	5	**6**	7	8	**9**	10	11	**12**	13	14	**15**	16	17	**18**	19	20	**21**	22	23	**24**
Counting in 3s	✓			✓			✓			✓			✓			✓			✓			✓			✓
Counting in 6s	✓						✓						✓						✓						✓

Maths | Modelling - Year 3: Multiplication and Division



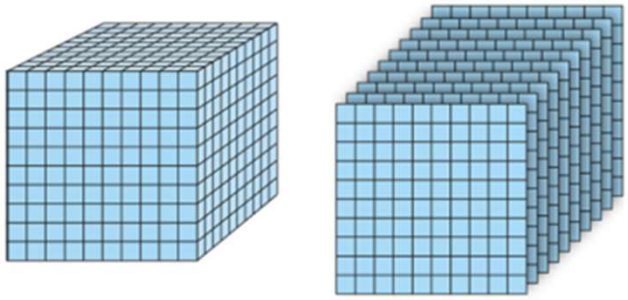
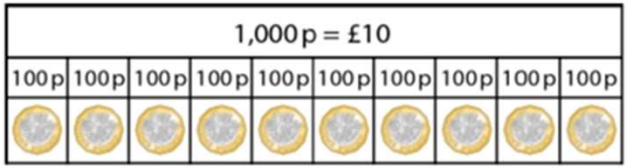
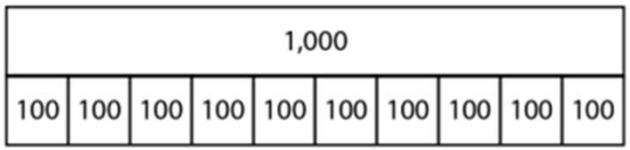
Number of boxes of 9 apples	Total number of apples
0	0
1	9
2	18
3	27
4	36
5	45
6	**54**

$0 \times 9 = 0$	$9 \times 0 = 0$
$1 \times 9 = 9$	$9 \times 1 = 9$
$2 \times 9 = 18$	$9 \times 2 = 18$
$3 \times 9 = 27$	$9 \times 3 = 27$
$4 \times 9 = 36$	$9 \times 4 = 36$
$5 \times 9 = 45$	$9 \times 5 = 45$
$\mathbf{6 \times 9 = 54}$	$\mathbf{9 \times 6 = 54}$

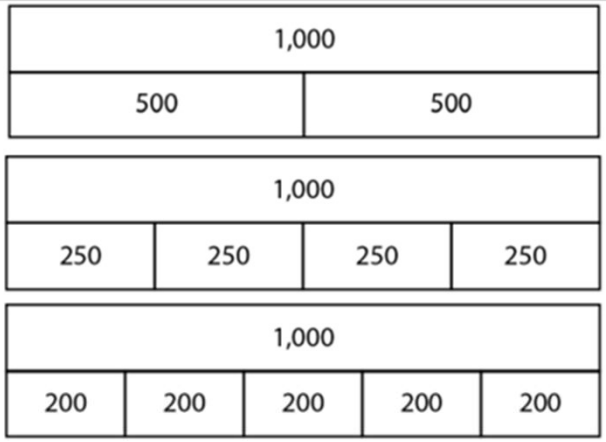
Maths | Modelling - Year 4: Addition and Subtraction

Year 4 – addition and subtraction

Application of Number Facts

	Concrete and Pictorial Representations	Abstract
Number facts to 1,000 (These are explored both as additive and multiplicative equations and applied within the range of strategies listed below)	Representing ten hundreds in 1,000: • Tens frame and 100 place-value counters  • Dienes  • Coins  	• Additive and multiplicative equations 1,000 = 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 + 100 1,000 = 10 × 100 1,000 = 100 × 10 1,000 ÷ 100 = 10 1,000 ÷ 10 = 100

Maths | Modelling - Year 4: Addition and Subtraction

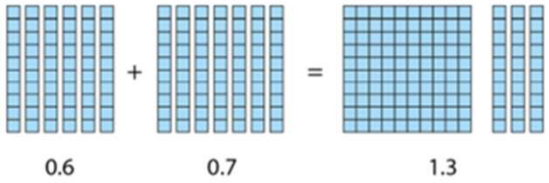


Mental Strategies with 4-digit numbers and decimals

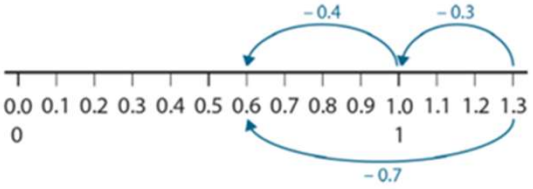
Each of the mental strategies taught in Year 3 are applied and explored within numbers in the thousands. This gives a chance for the children to be reminded of those strategies and gain familiarity in using them with increasing confidence while working with 4-digit numbers. These are further developed and extended once the children have learnt about decimals (tenths, hundredths and thousands). Examples of these strategies being used for decimals can be seen below.

See the Year 3 section above for an explanation and example of each mental strategy covered.

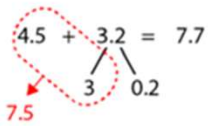
Dienes – example addition calculation:



Number line – example subtraction calculation:



Applying known strategies – partitioning numbers with tenths:



Maths | Modelling - Year 4: Multiplication and Division

Column Addition and subtraction

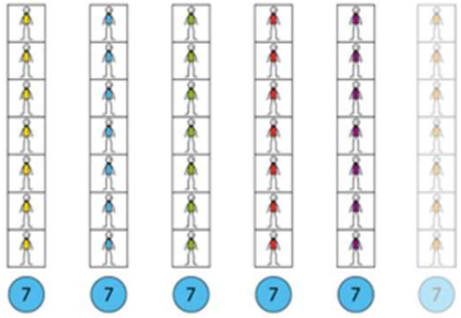
The column addition and subtraction algorithms taught in Y3 are extended and built upon in Y4 to include addition and subtraction of 4-digit numbers and decimals. For an explanation of these methods and how they are introduced, see the Y3 section above.		$\begin{array}{r} 13.2 \\ +\ 5.7 \\ \hline \\ \hline \end{array}$ $\quad$ $\begin{array}{r} 36.5 \\ -\ 2.3 \\ \hline \\ \hline \end{array}$

Year 4 – multiplication and division

Initially, the times table facts taught in Y3 are recapped to ensure these are secure before moving on to learn the 7x, 11x and 12x tables. This prepares them for a range of mental and written strategies for multiplication and division as well as for the statutory Multiplication Tables Check (a test given to all children in Year 4 to assess fluency of times tables recall).

Children in Year 4 are also taught a formal written method for multiplication and division: short multiplication and short division.

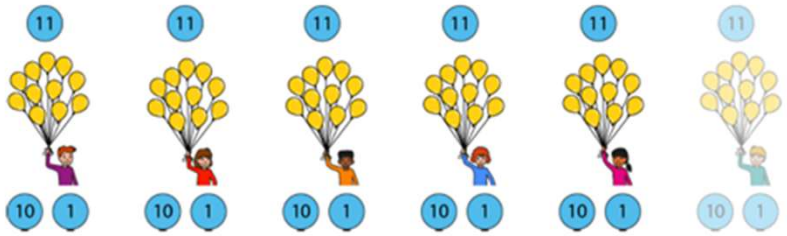
Number Facts

	Concrete and Pictorial Representations	Abstract
Times table number facts: 7x, 11x and 12x tables		

Number of netball teams	Total number of players
0	0
1	7
2	14
3	21
4	28
5	35
6	**42**

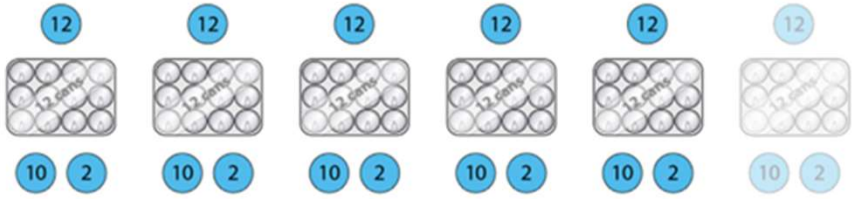
$0 \times 7 = 0$	$7 \times 0 = 0$
$1 \times 7 = 7$	$7 \times 1 = 7$
$2 \times 7 = 14$	$7 \times 2 = 14$
$3 \times 7 = 21$	$7 \times 3 = 21$
$4 \times 7 = 28$	$7 \times 4 = 28$
$5 \times 7 = 35$	$7 \times 5 = 35$
$\mathbf{6 \times 7 = 42}$	$\mathbf{7 \times 6 = 42}$

Maths | Modelling - Year 4: Multiplication and Division



Number of bunches of balloons	× 10	× 1	Total number of balloons (× 11)
0	0	0	0
1	10	1	11
2	20	2	22
3	30	3	33
4	40	4	44
5	50	5	55
6	**60**	**6**	**66**

0 × 11 = 0	11 × 0 = 0
1 × 11 = 11	11 × 1 = 11
2 × 11 = 22	11 × 2 = 22
3 × 11 = 33	11 × 3 = 33
4 × 11 = 44	11 × 4 = 44
5 × 11 = 55	11 × 5 = 55
6 × 11 = 66	**11 × 6 = 66**



0 × 12 = 0	12 × 0 = 0
1 × 12 = 12	12 × 1 = 12
2 × 12 = 24	12 × 2 = 24
3 × 12 = 36	12 × 3 = 36
4 × 12 = 48	12 × 4 = 48
5 × 12 = 60	12 × 5 = 60
6 × 12 = 72	**12 × 6 = 72**

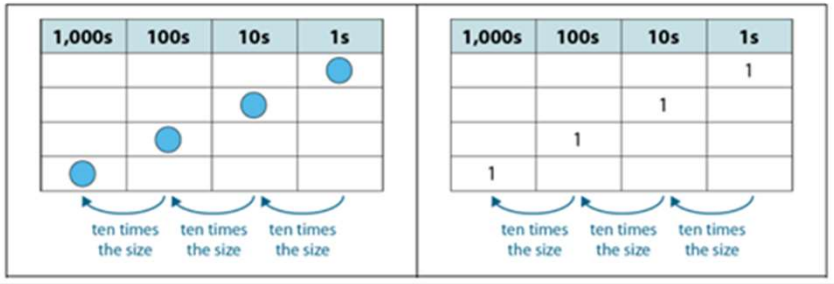
Maths | Modelling - Year 4: Multiplication and Division

Number of packs of cans	× 10	× 2	Total number of cans (× 12)
0	0	0	0
1	10	2	12
2	20	4	24
3	30	6	36
4	40	8	48
5	50	10	60
6	**60**	**12**	**72**

Multiplying and dividing by 10 and 100

	Concrete and Pictorial Representations	Abstract
The mental strategy for multiplying and dividing by 10 and 100 involves recognising the patterns within place value columns. For example, when a number is multiplied by 10, all of the digits move one place to the left (the number in the 1s moving to the 10s). This means that all of the digits will stay in the same order but will have a place holder in the 1s column).	Gattegno chart: × 10 ÷ 10	

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9



Maths | Modelling - Year 4: Multiplication and Division

Step 1 – move each of the digits one place to the left

1,000s	100s	10s	1s
		1	2
	1	2	

↓ × 10 *Think of '12' and make it ten times the size.*

ten times the size, ten times the size, ten times the size

Step 2 – write a '0' in the ones place

1,000s	100s	10s	1s
		1	2
	1	2	0

↓ × 10 *Think of '12' and make it ten times the size.*

ten times the size, ten times the size, ten times the size

Step 1 – move each of the digits two places to the left

1,000s	100s	10s	1s
		1	5
1	5		

↓ × 100 *Think of '15' and make it 100 times the size.*

100 times the size, 100 times the size

Step 2 – introduce zeros in the tens and ones places

1,000s	100s	10s	1s
		1	5
1	5	0	0

↓ × 100 *Think of '15' and make it 100 times the size.*

100 times the size, 100 times the size

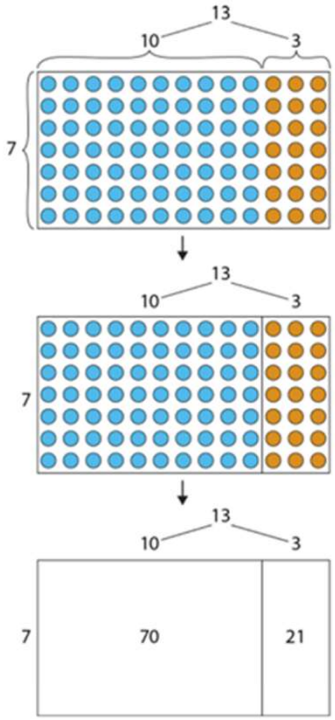
Ratio chart:

÷ 10	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
÷ 10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
	0	100	200	300	400	500	600	700	800	900	1,000	1,100	1,200	1,300	1,400	1,500

÷ 100

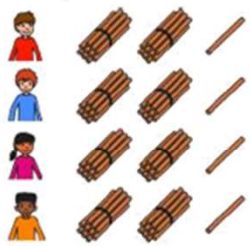
Maths | Modelling - Year 4: Multiplication and Division

Partitioning for multiplication

	Concrete and Pictorial Representations	Abstract
By applying knowledge of the distributive law (which means that if you split a number and multiply the split parts separately and add the separate answers together, you get the same answer as would get if you had multiplied the original number. The example here demonstrates this with 13 x 7. The 13 can be partitioned into a 10 and 3 with each of the partitions multiplied by 7. The products for those calculations can be added to find the final answer. **This strategy can be used both in written form and mentally, depending on the numbers involved and the strength of number fact knowledge.**		$13 \times 7 = 10 \times 7 + 3 \times 7$ $= 70 + 21$ $= 91$ $7 \times 13 = 7 \times 10 + 7 \times 3$ $= 70 + 21$ $= 91$

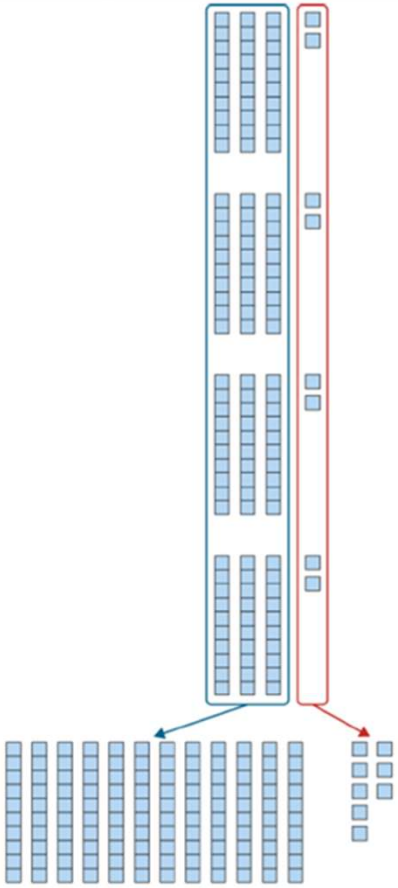
Maths | Modelling - Year 4: Multiplication and Division

Partitioning for division

	Concrete and Pictorial Representations	Abstract
Similar to multiplication, partitioning can be used to divide 2-digit numbers by single digit numbers. The example here shows how this can be done by splitting 84 into 8 tens and 4 ones which can each be divided by 4 and combined to reach the final answer. This is developed further later in KS2 when non-standard partitioning (splitting a number in a way other than by place value) can be used for efficient mental calculation. An example of this would be to solve 56 ÷ 4 by partitioning 56 into known multiples of 4 such as 40 and 16. Knowing that: 40 ÷ 4 = 10 and 16 ÷ 4 = 4 can allow the answer of 14 to efficiently calculated without the need for a written method.	*'Eighty-four sticks are shared equally between four children. How many sticks does each child get?'* 84 ÷ 4 = ?  Example solution reached by partitioning into tens and ones and dividing these separately. The quotients (answer for a division calculation) are then added.  8 tens ÷ 4 = 2 tens 4 ones ÷ 4 = 1 one 84 ÷ 4 = 21	

Maths | Modelling - Year 4: Multiplication and Division

Short multiplication

	Concrete and Pictorial Representations	Abstract
Short multiplication is a written method for multiplication taught as way to multiply a 2-digit number by a single digit number (such as 17 x 6). This is later extended to multiply 3 and 4-digit numbers by a single digit in Years 5 and 6. This method is initially introduced using physical resources such as dienes to represent what happens within the multiplication before moving to the written layout. It builds on their understanding of partitioning for multiplication explained above.		$32 \times 4 = 30 \times 4 + 2 \times 4$ $= 120 + 8$ • *'Three-tens-and-two-ones multiplied by four is equal to three tens multiplied by four and two ones multiplied by four.'* 3 tens × 4 = 12 tens 2 ones × 4 = 8 ones

Maths | Modelling - Year 4: Multiplication and Division

Example 1 – compact layout *with* place-value headings:

	10s	1s
	3	2
×		3
	9	6

- 3 × 2 ones = 6 ones
  'Write "6" in the ones column.'
- 3 × 3 tens = 9 tens
  'Write "9" in the tens column.'

Example 2 – compact layout *without* place-value headings:

	2	1
×		4
	8	4

- 4 × 1 one = 4 ones
  'Write "4" in the ones column.'
- 4 × 2 tens = 8 tens
  'Write "8" in the tens column.'

Maths | Modelling - Year 4: Multiplication and Division

When the product of one column is greater than 9, regrouping needs to be done. In this example, 3 x 4 gives the product of 12 ones which cannot fit in the one's column. Therefore, 10 ones are exchanged for 1 ten which is written below the tens column and the 2 is written in the ones column.

Example 1 – compact layout *with* place-value headings:

Step 1 – write the factors:

	10s	1s
	2	4
×		3

Step 2 – multiply the single-digit number by the ones and regroup:

	10s	1s
	2	4
×		3
		2
	1	

3 × 4 ones = 12 ones
= 1 ten + 2 ones

'Write "1" below the tens column and "2" in the ones column.'

Step 3 – multiply the single-digit number by the tens and add the tens from regrouping:

	10s	1s
	2	4
×		3
	7	2
	1	

3 × 2 tens = 6 tens

6 tens + 1 ten = 7 tens
'Write "7" in the tens column.'

Example 2 – compact layout *without* place-value headings:

	1	8
×		5
	9	0
	4	

- 5 × 8 ones = 40 ones; 40 ones = 4 tens and 0 ones
  'Write "4" below the tens column and '0' in the ones column.'
- 5 × 1 ten = 5 tens
  5 tens + 4 tens = 9 tens
  'Write "9" in the tens column.'

Maths | Modelling - Year 4: Multiplication and Division

Short division

	Concrete and Pictorial Representations	Abstract

Short division is a written method for division taught as way to divide a 2-digit number by a single digit number (such as 84 ÷ 4). This is later extended to divide by two-digit numbers in Years 5 and 6 but remains the most efficient written method for dividing by a single digit.

This method is initially introduced using physical resources such as dienes and pictorial representations to explore the method before moving to the written layout. It builds on their understanding of partitioning for division explained above.

Step 1 – write the divisor and dividend

10s 1s

4) 8 4

'Eighty-four divided by four.'

Step 2 – sharing the tens

10s 1s

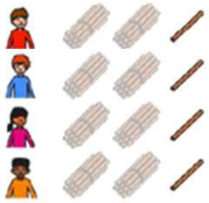
2

4) 8 4

8 tens ÷ 4 = 2 tens

'Eight tens divided by four is equal to two tens.'

Step 3 – sharing the ones



10s 1s

2 1

4) 8 4

8 tens ÷ 4 = 2 tens

4 ones ÷ 4 = 1 one

'Four ones divided by four is equal to one one.'

Summary

10s 1s

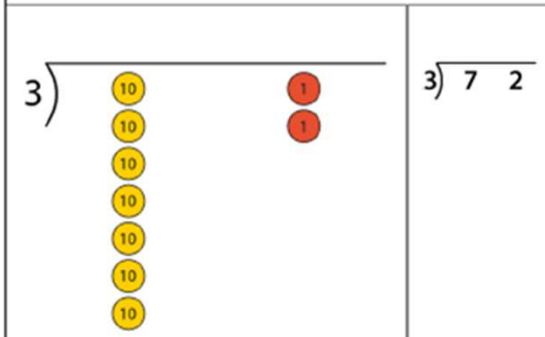
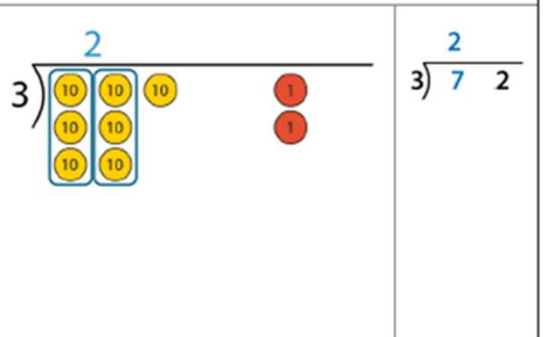
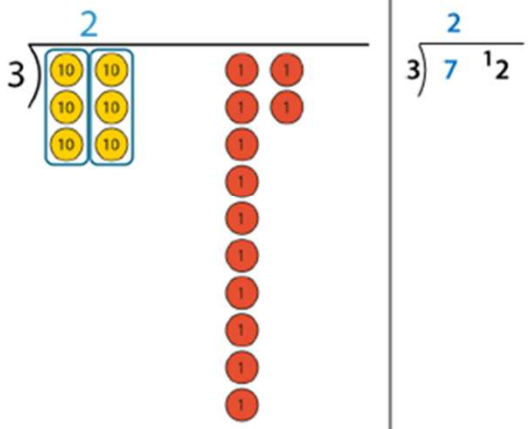
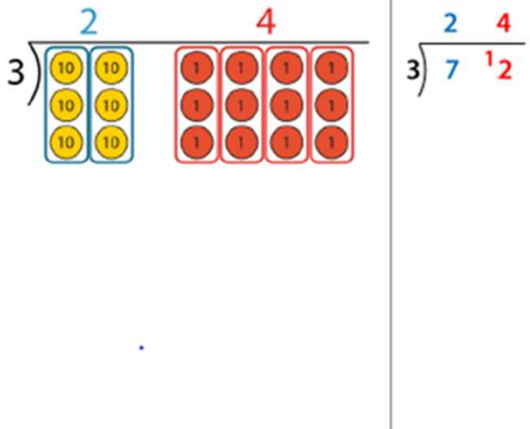
2 1

4) 8 4

'Each child gets twenty-one sticks.'

Maths | Modelling - Year 4: Multiplication and Division

When the division within a place value column leaves a remainder, exchanging is used.
In this example, when the 7 tens are divided by 3 there is a remainder of 1 ten. This is then exchanged for 10 ones giving a total of 12 ones to be divided next.

Step 1 – write the divisor and dividend	Step 2 – sharing the tens…
	
'Seventy-two divided by three.'	7 tens ÷ 3 = 2 tens r 1 ten *'Write "2" in the tens column…'*
Step 3 – …and exchanging	**Step 4 – sharing the ones**
	
1 ten = 10 ones *'…and write "1" to the left of the ones digit of the dividend to make twelve ones.'*	12 ones ÷ 3 = 4 ones *'Write "4" in the ones column.'*

Maths | Modelling - Year 5: Addition and Subtraction

Year 5 – addition and subtraction

Application of Number Facts

	Concrete and Pictorial Representations	Abstract
Number facts to 1,000,000 (These are explored both as additive and multiplicative equations and applied within the range of strategies listed below)		

10,000									
1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000

100,000									
10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000

10,000	
5,000	5,000

10,000 = 5,000 × 2

10,000 ÷ 2 = 5,000

10,000			
2,500	2,500	2,500	2,500

10,000 = 2,500 × 4

10,000 ÷ 4 = 2,500

10,000				
2,000	2,000	2,000	2,000	2,000

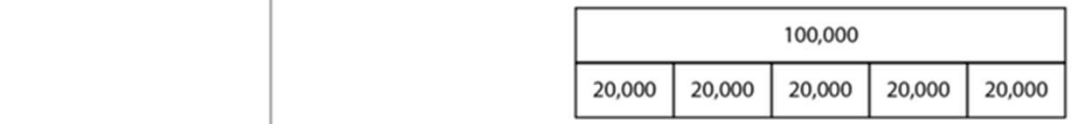
10,000 = 2,000 × 5

10,000 ÷ 5 = 2,000

Maths | Modelling - Year 5: Addition and Subtraction

100,000 = 50,000 × 2
100,000 ÷ 2 = 50,000

100,000 = 25,000 × 4
100,000 ÷ 4 = 25,000



100,000 = 20,000 × 5
100,000 ÷ 5 = 20,000

Mental Strategies with 5 and 6-digit numbers

Each of the mental strategies taught in Year 3 and used in Year 4 with 4 -digit numbers are applied and explored within numbers in the tens of thousands and hundreds of thousands. This gives a chance for the children to be reminded of those strategies and gain familiarity in using them with increasing confidence while working with 5 and 6-digit numbers.

See the Year 3 section above for an explanation and example of each mental strategy covered.

Column Addition and subtraction

The column addition and subtraction algorithms taught in Y3 and extended to include 4-digit numbers in Y4 are further extended into the tens of thousands and hundreds of thousands. Examples of these can be seen below.

For an explanation of these methods and how they are introduced, see the Y3 section above.

Maths | Modelling - Year 5: Addition and Subtraction

Column addition and subtraction:

- With place-value headings

	Thousands			Ones		
	100s	10s	1s	100s	10s	1s
	3	6	5	0	0	0
+	2	1	4	0	0	0
	5	7	9	0	0	0

- Without place-value headings

```
  3 6 5, 0 0 0
+ 2 1 4, 0 0 0
  ---------------
  5 7 9, 0 0 0
```

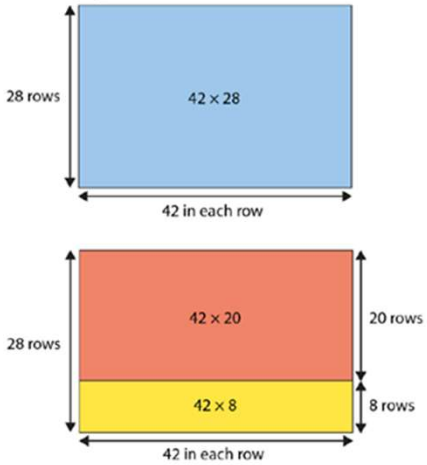
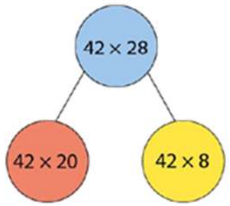
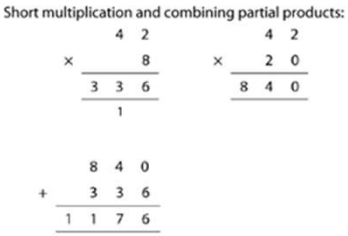
Maths | Modelling - Year 5: Multiplication and Division

Year 5 – multiplication and division

By the time they enter Year 5, children are expected to be able to confidently recall their times tables up to 12x12. This knowledge forms the foundation for learning more advanced methods and working with both larger numbers and with decimals.

Children in Year 5 are also taught a formal written method for multiplying and dividing by two digit numbers: long multiplication and long division.

Long Multiplication

	Concrete and Pictorial Representations	Abstract
Long multiplication is introduced by first using short multiplication by both the ones and tens separately. These answers (known as partial products) can then be added to find the product of the complete multiplication. This introduces the children to the fact that multiplication by a 2 digit number can be done by first multiplying by the ones, then the tens and these products then added.	Area model/grid:  Part–part–whole model: 	Short multiplication and combining partial products: 

Maths | Modelling - Year 5: Multiplication and Division

The separate short multiplication and addition steps are then combined within the long multiplication algorithm. To the right is shown the expanded layout. This most clearly shows each of the steps within the method. Children will quickly move from this expanded layout shown below once they are secure with the method.

Multiplication algorithm – expanded layout:

Step 1 – write the factors

	100s	10s	1s
		3	1
×		2	4

Step 2 – multiply the ones digit by the ones digit

	100s	10s	1s	
		3	1	
×		2	4	
			4	4 × 1 one = 4 ones

Step 3 – multiply the tens digit by the ones digit and regroup

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	4 × 3 tens = 12 tens = 1 hundred + 2 tens

Step 4 – place a zero to show that it's ten times the size

	100s	10s	1s
		3	1
×		2	4
	1	2	4
			0

Step 5 – multiply the ones digit by the tens digit

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	
		2	0	2 tens × 1 one = 2 tens

Step 6 – multiply the tens digit by the tens digit

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	
	6	2	0	2 tens × 3 tens = 6 hundreds

Step 7 – add the partial products

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	31 × 4
	6	2	0	31 × 20
	7	4	4	

Maths | Modelling - Year 5: Multiplication and Division

Multiplication algorithm – expanded layout:

	100s	10s	1s	
		3	1	
×		2	4	
	1	2	4	31 × 4
	6	2	0	31 × 20
	7	4	4	

Multiplication algorithm – compact layout:

		3	1
×		2	4
	1	2	4
	6	2	0
	7	4	4

Maths | Modelling - Year 5: Multiplication and Division

Step 4 – subtract to find the remainder

$$\begin{array}{r} 0\ 1 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1 \\ \hline 1\ 2 \end{array}$$

(1 ten × 31 = 31 tens)

43 tens – 31 tens = 12 tens

- *'Write "12" underneath the "31".'*

Step 5 – exchange tens for ones and combine with the existing ones

$$\begin{array}{r} 0\ 1 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1\ \downarrow \\ \hline 1\ 2\ 4 \end{array}$$

(1 ten × 31 = 31 tens)

12 tens = 120 ones

120 ones + 4 ones = 124 ones

- *'Write "4" after the "12".'*

Step 6 – divide the ones

$$\begin{array}{r} 0\ 1\ 4 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1 \\ \hline 1\ 2\ 4 \\ 1\ 2\ 4 \end{array}$$

(1 ten × 31 = 31 tens)

(4 ones × 31 = 124 ones)

124 ones ÷ 31 = 4 ones

(refer to the ratio chart)

- *'Write "4" in the ones column of the answer line and write "124" underneath the "124", aligning the digits.'*

Step 7 – subtract to show there is no remainder

$$\begin{array}{r} 0\ 1\ 4 \\ 31\overline{)4\ 3\ 4} \\ 3\ 1 \\ \hline 1\ 2\ 4 \\ 1\ 2\ 4 \\ \hline 0 \end{array}$$

(1 ten × 31 = 31 tens)

(4 ones × 31 = 124 ones)

124 ones – 124 ones = 0 ones

- *'Write "0" underneath the "31".'*

Maths | Modelling - Year 6: Addition and Subtraction

Year 6 – addition and subtraction

Application of Number Facts

	Concrete and Pictorial Representations	Abstract
Number facts to 100,000,000 (These are explored both as additive and multiplicative equations and applied within the range of strategies listed below)		

1,000,000	
500,000	500,000

$1{,}000{,}000 \div 2 = 500{,}000$

$1{,}000{,}000 \div 500{,}000 = 2$

$\frac{1}{2} \times 1{,}000{,}000 = 500{,}000$

$1{,}000{,}000 \times \frac{1}{2} = 500{,}000$

$2 \times 500{,}000 = 1{,}000{,}000$

$500{,}000 \times 2 = 1{,}000{,}000$

1,000,000			
250,000	250,000	250,000	250,000

$1{,}000{,}000 \div 4 = 250{,}000$

$1{,}000{,}000 \div 250{,}000 = 4$

$\frac{1}{4} \times 1{,}000{,}000 = 250{,}000$

$1{,}000{,}000 \times \frac{1}{4} = 250{,}000$

$4 \times 250{,}000 = 1{,}000{,}000$

$250{,}000 \times 4 = 1{,}000{,}000$

1,000,000				
200,000	200,000	200,000	200,000	200,000

$1{,}000{,}000 \div 5 = 200{,}000$

$1{,}000{,}000 \div 200{,}000 = 5$

$\frac{1}{5} \times 1{,}000{,}000 = 200{,}000$

$1{,}000{,}000 \times \frac{1}{5} = 200{,}000$

Maths | Modelling - Year 6: Addition and Subtraction

Mental Strategies with numbers in the millions

Each of the mental strategies taught in Year 3 and used in Year 4 and 5 with increasingly larger numbers are applied and explored within numbers in the millions. This gives a chance for the children to be reminded of those strategies and to explore how patterns within the small place values can also been seen in larger numbers (for example, that 4 x 250 = 1,000 and 4 x 250,000 = 1,000,000).

See the Year 3 section above for an explanation and example of each mental strategy covered.

Column Addition and subtraction

The column addition and subtraction algorithms taught in Y3 and used in Year 4 and 5 with increasingly larger numbers are further extended into the millions.

For an explanation of these methods and how they are introduced, see the Y3 section above.

Column addition:

$$\begin{array}{r} 643,801 \\ +\ 505,370 \\ \hline 1,149,171 \\ {\scriptstyle 1\ \ \ \ \ \ } \end{array}$$

Maths | Modelling - Year 6: Multiplication and Division

Long Division

	Concrete and Pictorial Representations	Abstract
The method for long division is taught to provide a formal written method when dividing by two-digit numbers that cannot be calculated mentally. Before attempting the written method, a ratio chart (frequently referred to as a 'What I Know' box of W.I.K.) must be created. This is a starter list of times tables for the divisor. It is not a complete list; it contains those that can be calculated quickly and acts as a starting point to work out the others if needed. The 2x, 4x, and 8x can be found by doubling, doubling and doubling again. The 10x can be found by using knowledge of place value and the 5x can be found by halving the 10x. The initial layout for long division is very similar to short division with the key difference being that the subtraction step is recorded, rather that being held mentally. This is to avoid errors which would likely occur if all of the steps of the division were held mentally.	Ratio chart: × 31 1 — 31 2 — 62 3 — 4 — 124 5 — 155 6 — 7 — 8 — 248 9 — 10 — 310	Step 1 – write the divisor, frame and dividend 31)4 3 4
	Step 2 – divide the hundreds 0 31)4 3 4 4 hundreds ÷ 31 = 0 hundreds r 4 hundreds • *'Write "0" in the hundreds column of the answer line.'*	Step 3 – exchange hundreds for tens, combine with the existing tens and divide… 0 1 31)4 3 4 3 1 (1 ten × 31 = 31 tens) 4 hundreds = 40 tens 40 tens + 3 tens = 43 tens 43 tens ÷ 31 = 1 ten and a remainder • *'Write "1" in the tens column of the answer line and write "31" underneath the "43".'*

STEM Faculty

It is our belief that it is vital for all children to question, find problems and most importantly find ways to solve those problems. Through the subjects of mathematics, science, design technology and computing, we aim to teach children to be interested in these concepts to allow them to help shape a better tomorrow.

Science

Design Technology

Computing